

INTER-GENERATIONAL DISTRIBUTION OF THE LIFE-CYCLE COST OF AN ENGINEERING FACILITY

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ABSTRACT

In decision making for civil engineering facilities, as well as other societal activities, the criteria for sustainability are inter-generational equity and optimality. Two challenging questions must be addressed in this context: How to compare the benefits and costs among different generations and how to compensate and adjust for the inhomogeneously distributed benefits and costs between the generations. To address and answer these questions for engineering facilities, first of all the temporal distribution of the life-cycle benefits must be assessed. To ensure optimality, the total life-cycle benefits for the facility must be maximized. In the present paper initially the normative criteria for sustainability are presented. Thereafter it is demonstrated how the criteria may be implemented for the purpose of optimization of structural design. The inter-generational distribution of benefits and the implications for sustainable decision-making are then illustrated by an example considering the optimal design of the concrete cover thickness of a RC structure subject to chloride-induced corrosion of the reinforcement.

KEYWORDS

Sustainability, discounting, life-cycle cost, chloride-induced corrosion, cover thickness.

1. INTRODUCTION

A significant amount of research has been devoted to life-cycle analysis for civil engineering facilities. In recognition of the significant uncertainties associated with the performance of structures over their service life, decision-theoretical approaches have been applied for the optimization of structural design, e.g., Rosenblueth and Mendoza (1971) and Rackwitz (2000). The developed methodological framework facilitates the optimization of the design of structures such that a balance is achieved

between the benefits achieved through the facility and the costs associated with design and construction, future costs of inspection and maintenance as well as costs associated with possible repairs, replacements and failures. Recently, life-cycle analysis has been utilized to enhance a sustainable development of the built environment, e.g. Rackwitz et al. (2005), Faber and Rackwitz (2004), Nishijima et al. (2004) and Nishijima et al. (2005). In this context, focus is shifted from the facilities to a sequence of decision makers and stake holders, each of which represents a subsequent generation that benefits from the facility while paying the costs of maintenance, repair, replacement and other adverse consequences. Although life-cycle analysis is well advanced in the civil engineering field and has been applied within the context of sustainability, less attention has been paid to the distribution of costs over time. This distribution is essential, since it allows for assessing the burden of each generation, and thus indicates the necessity for an inter-generational compensation when the aggregation of benefits and costs is not uniformly distributed over time.

The present paper initially formulates the criteria for sustainability and thereafter sets up a multi-decision-maker framework for inter-generational sustainable decision making. As it will be discussed this framework may also provide a useful basis in any intra-generational context for organizations involved in decision making concerning activities with life times significantly exceeding the budgeting periods or the life time of the individuals responsible for the decision making within the organization. The optimization of structural design using the suggested framework is illustrated by an example considering the optimal design of the cover thickness for a RC structure subject to chloride-induced corrosion. Finally, the temporal distribution of the life-cycle costs is explicitly assessed, clearly illustrating how the benefits and costs are unevenly distributed over the generations.

2. MULTI-DECISION-MAKERS AND CRITERIA FOR SUSTAINABILITY

Sustainability is interpreted in accordance with the Rio convention in 1992, following the report by Brundtland (1987). To facilitate sustainable decision making, two criteria are provided: 1) *inter-generational equity* and 2) *optimality*. *Inter-generational equity* dictates equal treatment of the present and all future generations. *Optimality* can be interpreted as the maximization of an idealized utility function, considering all generations and their preferences. These two criteria are strongly interrelated and this must be taken into account in the decision making. In order to set up the utility function aggregating the benefits and costs for all generations, the equal treatment of the individual generations in accordance with the *inter-generational equity* criterion is required. Once *optimality* is obtained by maximizing the idealized utility function, the temporal in-homogeneity of the utilities among the different generations must be reconsidered to ensure *inter-generational equity*.

Basically any kind of activity at present has consequences for the future in terms of benefits and costs. The benefits and costs may not necessarily be expressed in monetary terms and there are controversial discussions on whether all societal and environmental consequences can be measured comprehensively in monetary terms, as discussed in Turner (1992) and Ayres et al. (1998). However, in the present paper, benefits and costs are assumed to be represented by monetary values for the convenience of discussion. The temporal distribution of consequences associated with

different activities differ significantly, however, it is difficult to identify activities which do not have some effect for the future generations. In case of exploitation of natural resources the benefit is more or less immediate – but the resources exploited are no longer available for future generations. In case of disposal of toxic waste the situation is much the same – the benefit is achieved by the present generation but the potential adverse consequences are likely to be transferred to future generations. Sustainability is an issue which always has to be kept in mind.

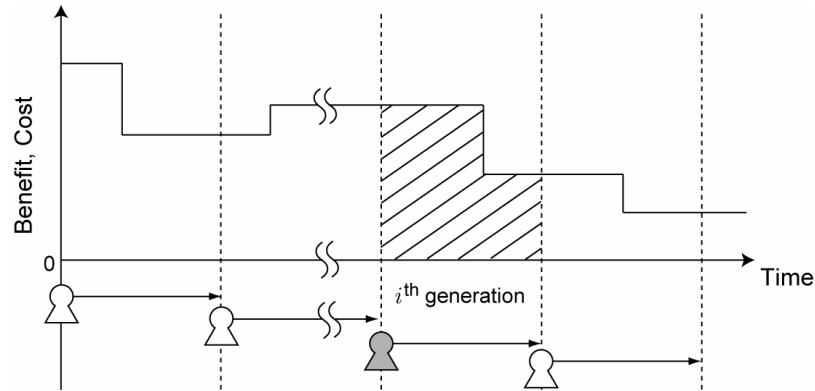


Figure 1. Schematic distribution of benefit or cost over time.

The schematic benefit or cost path is illustrated in Figure 1. A sequence of decision makers is assumed along with the time, each representing one generation. Since each generation considers the benefits and costs and makes decisions from its point of view, an explicit modeling of the different subsequent decision makers is indispensable, especially when the pure time preference or loss of life are considered in the utility function.

The benefits and the costs illustrated in Figure 1 correspond to the gross values at each point in time, i.e., they are not discounted. The i^{th} generation enjoys the benefit or carries the cost of the hatched area. Since this is the gross value, the same values at different points in time do not necessarily have the same perceived influence to different generations, mainly because of the economic growth. Therefore, benefits and costs should be discounted by the economic growth to ensure the equal treatment between generations in accordance with the inter-generational equity. Taking into account the economic growth and disregarding the effect of overlapping generations, the total utility aggregating benefits and costs can be expressed as:

$$U = \sum_{i=1}^{\infty} \delta(t_i) U_i \quad (1)$$

where U is the total utility for all generations, $\delta(\cdot)$ is the discounting factor representing economic growth and U_i is the utility for i^{th} generation which begins at $t = t_i$. Extension of Equation (1) to cover also the case of overlapping generations may be performed as shown in Bayer and Cansier (1999), Bayer (2003) and Rackwitz et al. (2005), however, the effect of this is of minor importance for the overall life-cycle benefit assessment. When decision making is subject to uncertainty, the utilities in Equation (1) should be interpreted as the expected utilities. The utility for the i^{th} generation may be written as:

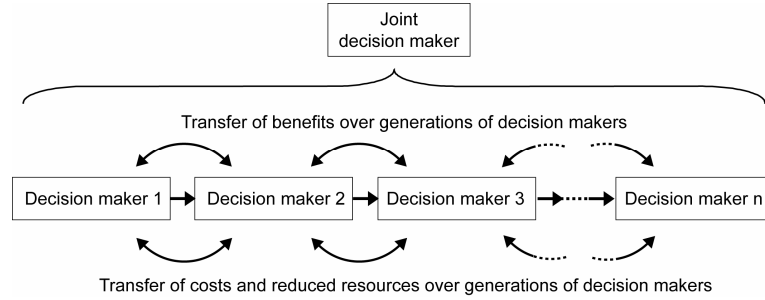


Figure 2. Transfer of benefits, costs and resources.

$$U_i = \int_{t_i}^{t_{i+1}} u(t)\gamma(t-t_i)dt \quad (2)$$

where $u(\cdot)$ is the utility per unit time and $\gamma(\cdot)$ is the discounting factor within one generation. The utility within one generation may be discounted by pure time preference as well as by economic growth, thus

$$\gamma(t) = \delta(t)\rho(t) \quad (3)$$

where $\rho(\cdot)$ is the discounting factor representing pure time preference. Note that the discounting factor is related to the discount rate, e.g., for $\delta(\cdot)$ as:

$$\delta(t) = \exp(-\delta t) \quad (4)$$

where δ is the discount rate per unit time.

Each decision in regard to a civil engineering facility results in one specific temporal distribution of expected utility and thus enables the calculation of the total utility according to Equation (1). To comply with the second criterion for sustainability, i.e., optimality, the total utility must be maximized, which in the case where the benefit function does not depend directly on the decision corresponds to a minimization of the total cost. However, even if the maximization is performed under consideration of inter-generational equity in terms of proper discounting as applied in Equations (1)-(3), it does not necessarily imply that each generation obtains the same utility from the facility, as illustrated in Figure 1. It is unlikely that each single activity optimized in the above sense results in a uniform distribution of the utility among the current and all future generations. Therefore, the transfer of the benefits in terms of, for instance, man-made capital or natural resources is essential to achieve inter-generational equity, see Figure 2. The distribution of costs over time provides the basic information required to achieve inter-generational equity, enabling a comparison and a compensation between the generations through societal activities which are not necessarily within the civil engineering field.

3. EQUIVALENT SUSTAINABLE DISCOUNT RATE

Classical life-cycle cost analysis approaches the discounting problem from the perspective of the anticipated duration of the considered activity, e.g. the anticipated service life when a given structure is considered. Furthermore, decision making in classical life-cycle analysis takes basis in a utility modeling where only the preferences of the present generation are directly accounted for. This includes also the

aspects of valuation of future benefits and costs through discounting. For a given activity it is possible to assess a discount rate which if applied in a classical life-cycle

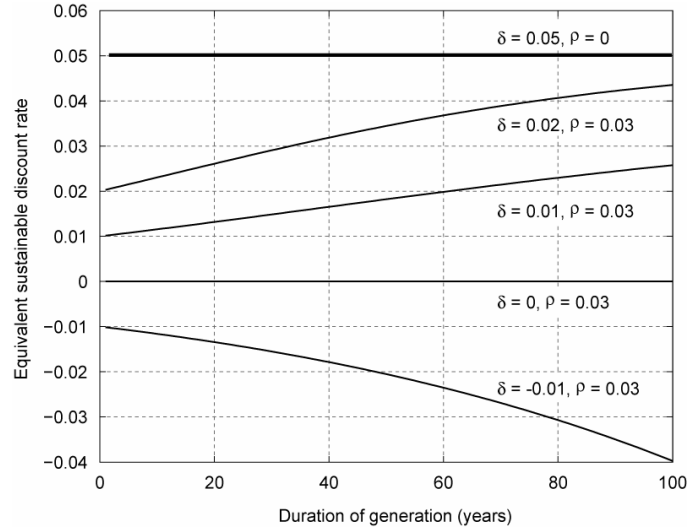


Figure 3. Equivalent sustainable discount rate γ^* for the case of constant utility per unit time.

analysis yields the same total expected utility as resulting from the proposed multi-decision-maker framework (Equations (1)-(3)). This discount rate is denoted the equivalent sustainable discount rate γ^* by:

$$\int_0^{\infty} e^{-\gamma^* t} u(t) dt = \sum_{n=1}^{\infty} e^{-\delta t_n} \int_{t_n}^{t_{n+1}} e^{-\gamma(t-t_n)} u(t) dt \quad (5)$$

where $u(t)$ is the (expected) utility per unit time at time t . The equivalent sustainable discount rate may be interpreted as the one which, if applied to a decision problem with the classical one-decision-maker perspective, yields the same total expected utility as when the decision problem is analyzed from the multi-decision-maker perspective. In general, it is not possible to obtain an analytical expression for γ^* . However, in the case where consequences are invariant at any time, the durations of generations $\tau = t_{n+1} - t_n$ ($n = 1, 2, 3, \dots$) are constant and the occurrences of events associated with consequences follow a stationary Poisson process, the equivalent sustainable discount rate is given as follows:

$$\gamma^* = \frac{1 - e^{-\delta \tau}}{1 - e^{-\gamma \tau}} \gamma \quad (6)$$

where δ is the discount rate per unit time by economic growth, ρ is the discount rate per unit time by pure time preference and $\gamma = \delta + \rho$, see Faber and Nishijima (2004). The equivalent sustainable discount rates for several cases are illustrated in Figure 3, where for ρ kept constant at 3% per year or 0% per year for comparison, the equivalent sustainable discount rates are given as functions of the duration of the generation τ for several values of δ . The equivalent sustainable discount rate γ^* is smaller than the total discount rate γ , except for the case where $\rho = 0$. If the discount rate consists only of pure time preference ($\delta = 0$), the equivalent sustainable discount rate is zero, i.e., within the classical framework, the benefits and costs should not be discounted at all to obtain the same utility function as with the multi-decision-maker

framework. If the discount rate by pure time preference is set equal to zero and the discount rate by economic growth is set to equal to 5%, the equivalent sustainable discount rate is equal to 5%, regardless of the duration of the generation. This means that if the discount rate is only due to economic growth, the multi-decision-maker framework is identical to the classical framework. In general, the discount rate which has been applied so far in the classical framework is too large, i.e., is leading to non-optimal solutions from the view-point of sustainability.

4. EXAMPLE

Optimal life-cycle cost based design of the concrete cover thickness of a RC structure subject to chloride-induced corrosion of the reinforcement is considered. The intended service life time is assumed to be infinite, meaning that the desired function of the structure is unlimited in time. The applied probabilistic modeling of the degradation over time is included in Annex A for simple reference and more details are provided in Faber et al. (2005). The expected life-cycle costs are assumed to consist of the initial costs C_I , the expected repair costs $E[C_R]$ and the expected failure costs $E[C_F]$, which all depend on the optimization variable d_{nom} , i.e. the concrete cover thickness. It is assumed that visual inspections are made every $\Delta t_I = 5\text{yr}$ and that an indication of visible corrosion automatically triggers a repair. In accordance with the renewal-theoretical approach outlined in Faber and Rackwitz (2004), it is assumed that in case the structure fails, it is reconstructed. Following a repair or a reconstruction, the structure is assumed brought back to its original state, i.e., described using the same probabilistic model as a new structure. The realization of the structure after repair or reconstruction is assumed to be independent from previous structures. Furthermore, inspections are modeled as being perfect, i.e., visible corrosion is detected with probability 1 at an inspection. The costs of initial design, repairs and failures are modeled as:

$$C_I = (1 + a_I d_{nom}) C_0 \quad (7)$$

$$C_R = a_R C_I \quad (8)$$

$$C_F = a_F C_I \quad (9)$$

with parameter values in Table 1, where also the assumed discount rates are summarized. The initial cost C_I is assumed to consist of a fixed cost and the cost depending on the cover thickness, and the repair cost C_R and the failure cost C_F are assumed to be proportional to the initial cost.

Table 1. Cost and discount model.

Discount rate for time preference: r	3% per year
Discount rate for economic growth: d	2% per year
Normalizing cost C_0	1
Cost ratio for cover thickness a_I	0.002
Coefficient of repair cost a_R	0.5
Coefficient of failure cost a_F	5

4.1. Cost distribution over time

In order to calculate the distribution of life-cycle costs over time, an efficient algorithm is required, since the number of branches in the decision tree develops exponentially with time. In Nishijima et al. (2004), these costs are calculated by using a recursive formulation; in the following, a different recursive formulation is provided which facilitates the explicit calculation of the expected cost of repair and failure at each point in time. After specifying the decision rule, which defines in which situations a repair is made, the probability of repair $q_R(t)$ and the probability of failure $q_F(t)$ at time t ($t = 1\text{yr}, 2\text{yr}, 3\text{yr}, \dots$) for a given realization of the structure are readily available, see e.g. Faber et al. (2005), Nishijima et al. (2004, 2005). In accordance with the above, the decision rule adopted in this example is that the structure is repaired if and only if corrosion is visibly observed at the inspection. Whether or not this decision rule is optimal is beyond the scope of this paper, which focuses on the design optimization. However, by consideration of the deterioration model and the possible actions it is easily seen that, for the present example, there are only a few reasonable alternative rules. When optimizing the inspection/maintenance strategy, these alternatives can be compared and the one leading to minimal costs can be selected, e.g., Straub (2004). According to the probabilistic model in Annex A, $q_R(t)$ and $q_F(t)$ are estimated by Monte Carlo simulation with 10^6 samples for each cover thickness, see Figure 4 and 5. Note that the probability of repair can be different from zero only at $t = i\Delta t_i$ ($i = 1, 2, 3, \dots$), since the repairs are associated with inspections which are made at intervals of $\Delta t_i = 5\text{yr}$; failure can occur in any year, but its probability is increasing with time and thus more likely to occur when approaching the inspections. With the probabilities $q_R(t)$ and $q_F(t)$, the probability of repair $P_R(t)$ and the probability of failure $P_F(t)$ at time t are calculated based on the renewal theory (see, e.g., Feller (1966) in general and Rackwitz (2000) considering applications to civil engineering facilities) as:

$$P_R(t) = q_R(t) + \sum_{s=1}^{t-1} (P_R(s) + P_F(s)) \cdot q_R(t-s) \quad (10)$$

$$P_F(t) = q_F(t) + \sum_{s=1}^{t-1} (P_R(s) + P_F(s)) \cdot q_F(t-s) \quad (11)$$

for $t = 2\text{yr}, 3\text{yr}, 4\text{yr}, \dots$ and

$$P_R(1\text{yr}) = q_R(1\text{yr}) \quad (12)$$

$$P_F(1\text{yr}) = q_F(1\text{yr}) \quad (13)$$

for $t = 1\text{yr}$.

The recursive formulations Equations (10)-(13) are obtained as follows. The set of possible different events leading to a repair at time t can be split into subsets: These subsets are differentiated by the time of the last repair or reconstruction, which can occur at times $t-1\text{yr}$, $t-2\text{yr}$, etc. until 0yr ; the latter corresponding to the case where no repair or reconstruction has been performed previously. The probability of failure at time t is obtained analogously. As the decision rule just specifies $q_R(t)$ and $q_F(t)$, this recursive formulation can be applied for any kind of decision rule, as long

as the structure is repaired at some point in time and reconstructed after failure, resulting in identical but stochastically independent structures.

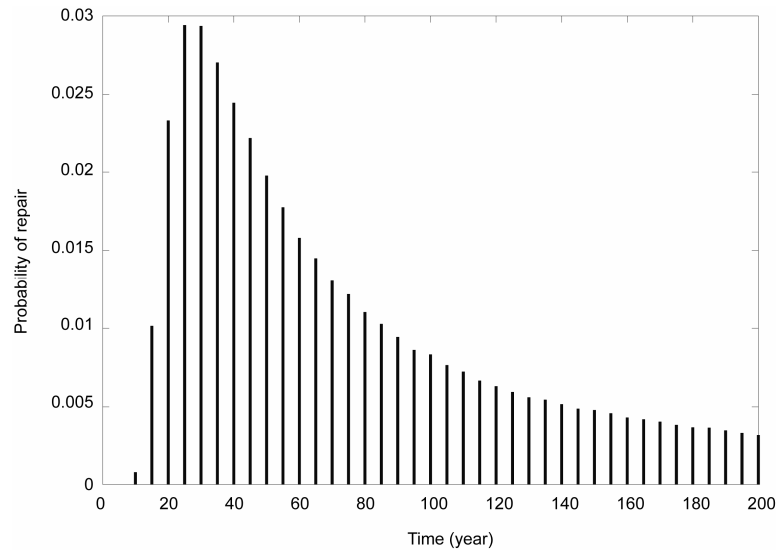


Figure 4. Probability of repair $q_R(t)$ at time t for a given realization of the structure (cover thickness = 50 mm).

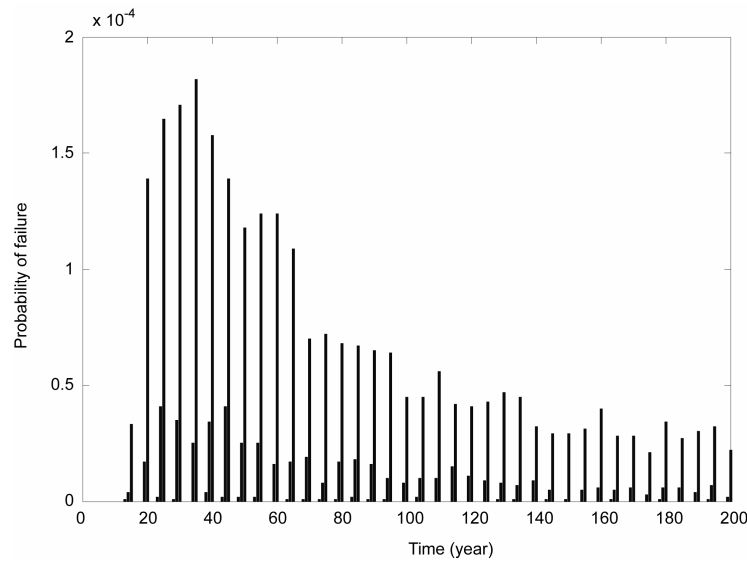


Figure 5. Probability of failure $q_F(t)$ at time t for a given realization of the structure (cover thickness = 50 mm).

Once the probability of repair and the probability of failure at each point in time are obtained, the calculation of the expected costs is straightforward. Repair and reconstruction after failure can be carried out at each inspection time, the inspection interval being 5 years. Figure 6 shows the distribution of costs over time for several cover thicknesses. These costs are not discounted. The expected cost for each point in time consists of the expected repair costs and the expected failure costs. The expected failure costs are much smaller than the expected repair costs in the present example, that is why the expected total costs are close to the expected repair costs.

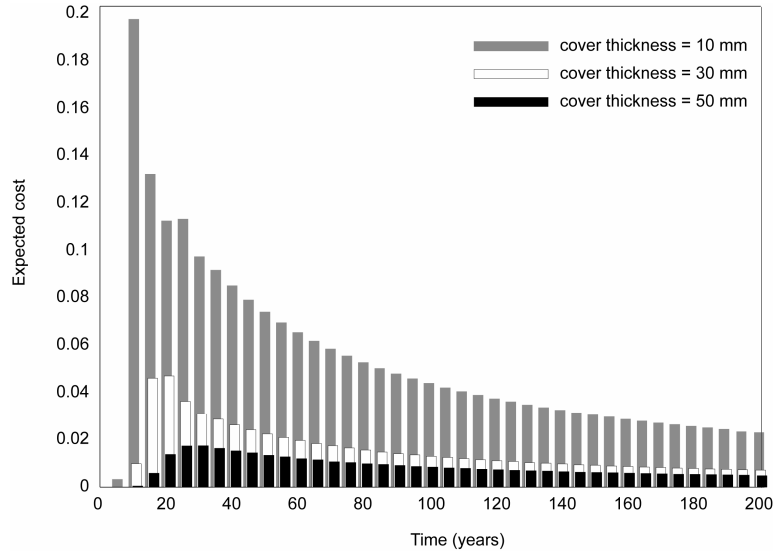


Figure 6. Temporal distribution of expected costs (at 5 year intervals), not discounted.

The (non-discounted) expected costs decrease with time for all cases in Figure 6. This tendency is due to the fact that the failure rate, which is the probability of failure per unit time conditional on survival up to time t , is decreasing with time for the considered deterioration mechanism. When the structure performs poorly (i.e., the realizations of the random variables are unfavorable), it will be repaired or reconstructed already after a few years. After each repair or reconstruction, the new structures are identical but stochastically independent of the old ones. A structure with an initially bad performance will thus eventually be replaced by one with a good performance. The expected value of the performance of the structure is therefore increasing with time and the expected costs of failures and repairs are decreasing. It should be realized that this tendency depends strongly on the assumed dependency between subsequent realizations of the structure as well as the characteristics of the failure rate function.

4.2. Optimization of the concrete cover thickness

Taking basis in the multi-decision-maker framework presented in the previous section, the total expected costs to be minimized are calculated for each decision alternative (i.e., for different cover thicknesses). For this example the total expected costs reduce to:

$$\begin{aligned}
 -E[U(d_{nom})] &= E[C(d_{nom})] \\
 &= C_I(d_{nom}) + \sum_{i=1}^{\infty} \delta(t_i) \sum_{j=1}^{\tau/\Delta t_I} (E[C_{R,t_i+j\Delta t_I}(d_{nom})] + E[C_{F,t_i+j\Delta t_I}(d_{nom})]) \gamma(j\Delta t_I) \quad (14)
 \end{aligned}$$

which should be minimized. $C_{R,t}$ and $C_{F,t}$ are the costs of repair and failure at time t respectively. Since different discount rates are applied within the generations and between the generations, the duration of each generation τ must be specified. Figure 7 shows optimal cover thicknesses for different values of the durations of the generations. With increasing duration of generations, the optimal cover thickness becomes smaller. This is because the “equivalent sustainable discount rate” becomes larger as the duration of the generation becomes longer, see Equation (6) and Figure 3,

and consequences in the future are, therefore, valued less. The case where the duration of a generation is infinite corresponds to the classical life-cycle analysis where only one decision maker is assumed. As observed in Figure 7, the optimal cover thickness varies significantly with the duration of the generations, pointing to the importance of considering the problem from the viewpoint of the multi-decision-makers.

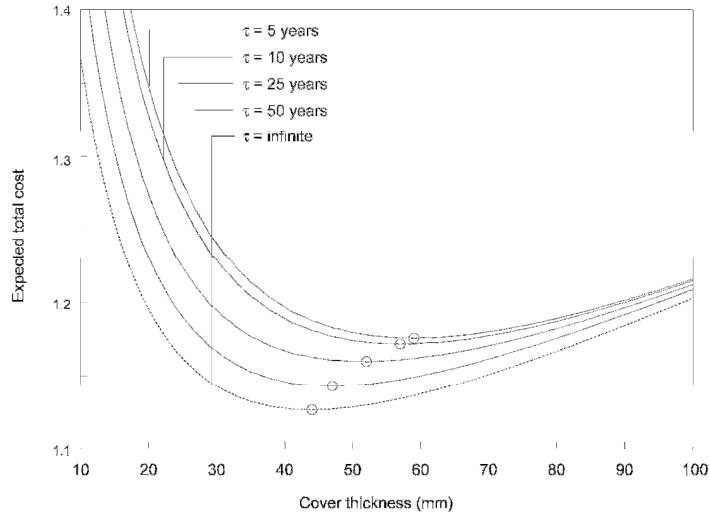


Figure 7. Optimal cover thickness for several durations of generation τ .

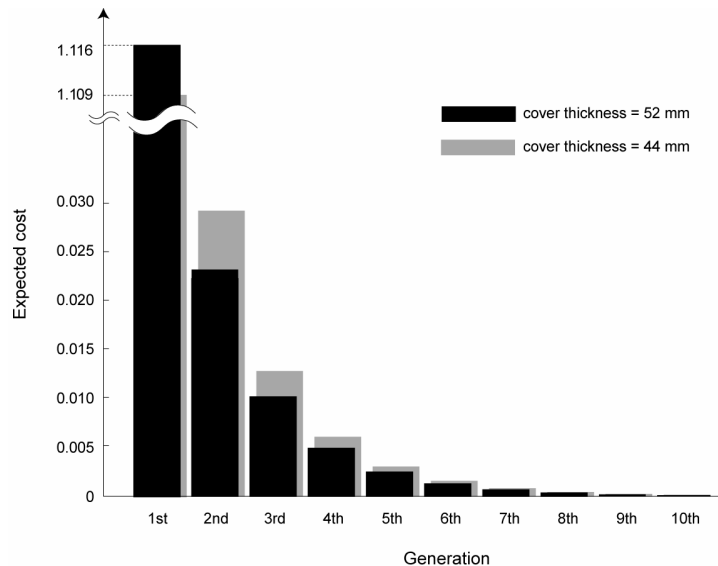


Figure 8. Discounted expected costs for each generation (with a duration 25 years).

In the following, the duration of a generation is assumed to be 25 years. When applying the multi-decision-maker framework, the optimal cover thickness is 52mm, in accordance with Figure 7. By applying the classical framework (the infinite duration of generations in Figure 7), the optimum is at 44mm. Figure 8 shows the corresponding expected costs with time. These costs are discounted to time $t = 0$, therefore, it is possible to compare these costs with each other. Within the classical framework the first generation pays less and the future generations pay more than within the multi-decision-maker framework. This is due to the fact that the classical

framework weighs values in the future less than the multi-decision-maker framework through the relatively higher discount rate.

5. DISCUSSION

Figure 8 clearly shows the inhomogeneous distribution of costs among the generations. In particular the first generation pays much more than all following generations. In order to comply with the first criterion for sustainability, *inter-generational equity*, the temporal differences must be compensated by other means (e.g., by transferring the benefits on capital stocks and natural resources). Such compensation is beyond the scope of the analysis as presented in this paper, as it requires that all societal activities must be considered simultaneously within the multi-decision-maker framework. In this context it is reminded that, although in the presented example it is the first generation which pays most, many societal activities have large consequences in the future while only the current generation directly benefits from them.

The presented framework can be extended to portfolios of structures, which are distributed over time and space. The optimization of design and maintenance activities is performed in analogy to the case of the individual structure, but to ensure *inter-generational equity* through compensation, it is required to consider the cost distribution over time for all structures simultaneously.

The analysis presented here ensures that the second criterion of sustainability, *optimality*, is fulfilled in such a way that it is consistent with the first criterion. It seems paradoxical at first that by consideration of multi-decision-makers (which is required by the *inter-generational equity* criterion), the optimal design which fulfills the *optimality* criterion leads to an even more inhomogeneous distribution of costs among generations. For this reason it is crucial that the issue of compensation between the generations is also addressed.

The presented multi-decision-maker framework provides an analytical approach to the consideration of the preferences of all generations involved in the life-cycle of engineering structures. It allows for the assessment of the effect of postponing costs to the future through the use of large interest rates, which is a common tendency in societal decision making. In order to be sustainable, the equivalent sustainable discount rate presented in this paper must be applied.

Finally, it is important to note that whereas the present paper specifically addresses the problem of sustainable decision making in an inter-generational context the developed framework also may be valuable for the decision making in intra-generational contexts involving several decision makers and stakeholders as well as budgets over time. This is the situation when decision making is considered in organizations which are responsible for the design, construction and operation of engineering facilities such as high-way agencies. In such organizations both budgets as well as the persons involved in the decision making have a substantially shorter life time than the facilities they are responsible for. The multi-decision-maker framework may serve to set guidelines or rules for the decision making in such contexts, to help avoid decisions which for the fulfillment of preferences of individuals may yield a short term benefit but from an overall life-cycle perspective induce economical losses

for the organization. Furthermore, the framework can be utilized as a rational basis for long term budgeting.

6. CONCLUSIONS

It is demonstrated how the inter-generational distribution of the life-cycle cost of an engineering facility can be assessed. This is of importance for ensuring sustainability of the facility, whereby the considered criteria for sustainability are *inter-generational equity* and *optimality*. It is shown how decisions regarding an engineering facility must be optimized in order to comply with these criteria and it is outlined that the results of the optimization may be used as a basis for a broader discussion regarding inter-generational equity taking into account all kinds of societal activities. Finally, it is highlighted that the developed framework also may provide a useful basis in any intra-generational context for organizations involved in decision making concerning activities with life times significantly exceeding the budgeting periods or the life time of the individuals responsible for the decision making within the organization.

The developed decision framework is illustrated by the optimization of the design of a RC structure subject to chloride-induced corrosion and is found to have a significant effect on the optimal design.

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ANNEX A

For easy reference, the applied probabilistic model for deterioration of concrete structures subject to chloride-induced corrosion is presented in the following. The modeling corresponds to DuraCrete (2000) and here follows Faber et al. (2004), where additional details of the models are described.

Corrosion initiates at the reinforcement, when the chloride concentration has reached the critical chloride concentration C_{cr} . The ingress of chlorides in the concrete is described by Fick’s second law of diffusion. Based on this model, the random variable T_I representing the time until corrosion initiation is calculated as:

$$T_I = \left(\frac{d^2}{4k_e k_i k_c D_0 (t_0)^n} \left[\operatorname{erf}^{-1} \left(1 - \frac{C_{cr}}{A_{C_s} \cdot (w/c) + \epsilon_{C_s}} \right) \right]^2 \right)^{\frac{1}{1-n}} \quad (a1)$$

The parameters of the model are given in Table A1.

The time until visible corrosion, corresponding to minor cracking and coloring of the concrete surface, can be determined based on experience. By adding the propagation

time T_p to the initiation time T_I , the limit state function for visible corrosion is written as:

$$g_{VC}(t) = X_I T_I + T_p - t \quad (a2)$$

The time between visible corrosion and failure is, for illustrative purposes, represented by the time T_p . The limit state function for failure is thus:

$$g_F(t) = X_I T_I + T_p + T_{p2} - t \quad (a3)$$

Note that the model does not account for the dependency between the propagation time T_{p2} and the environmental parameters or the cover thickness.

The values of the distribution parameters for the random variables in Equations (a1) to (a3) can be obtained as functions of indicators, see Faber et al. (2004). For the considered example, they are stated in Table A1. These values are representative for a concrete with ordinary Portland cement in a splash environment.

The probabilities of the events visible corrosion and failure can be obtained by e.g. Structural Reliability Analysis (SRA) or simulation techniques.

Table A1. Example parameters for the deterioration model.

Parameter	Description	Distribution	Dimension	m	s	A	B
d	Cover thickness	Lognormal	mm	d_{nom}	$0.3 \times d_{nom}$		
k_e	Environmental factor	Gamma	-	0.924	0.155		
k_c	Curing factor	Beta	-	0.8	0.1	0.4	1.0
k_I	Test factor	Deterministic	-	1.0	-		
D_0	Diffusion coef.	Normal	mm ² /yr	220.9	25.4		
t_0	Reference period	Deterministic	yr	0.077	-		
n	Age factor	Beta	-	0.362	0.245	0	0.98
C_{cr}	Critical chloride concentration	Normal	*	0.8	0.1		
w/c	Water/cement ratio	Deterministic	-	0.40	-		
A_{C_s}	Chloride surface concentration factor	Normal	*	7.758	1.36		
e_{C_s}	Chloride surface concentration factor	Normal	*	0	1.105		
X_I	Model uncertainty	Lognormal	-	1.0	0.05		
T_p	Propagation time	Lognormal	yr	7.5	1.9		
T_{p2}	Propagation time	Lognormal	yr	10.0	4.0		

* Mass-% of binder