A framework for the asset integrity management of large deteriorating concrete structures

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Many concrete structures can be considered as large parts of engineering systems. For these, realistic modelling of deterioration and the planning of inspection and maintenance activities requires explicit consideration of the interdependencies among the individual elements of the structure. Ageing of the infrastructure has become increasingly recognized by engineers and decision makers as being important, yet relatively few research projects have addressed this topic to date. The present paper provides an overview of the decision problems related to the planning of inspection, monitoring and maintenance activities in large engineering systems. This is followed by the presentation of a model framework for the representation of spatial and temporal variability of deterioration, illustrated by consideration of chloride-induced corrosion of the reinforcement in concrete structures. The framework is computationally highly efficient and facilitates the consistent, quantitative modelling of the risks from deterioration for the entire system at all points in time. Finally, it is demonstrated how the framework can be applied to optimize the planning of inspection and maintenance activities for large concrete structures.

Keywords: Inspection; Corrosion; Bayesian updating; Risk; Optimization; System

1. Introduction

Many past publications address the issue of life-cycle costing and optimization of control and maintenance activities for deteriorating structures, see Faber (2000), Onoufriou and Frangopol (2002), Ellingwood (2005) and Moan (2005), for an overview. Control actions include inspection and monitoring of the structure. Many of these approaches explicitly account for the various types of uncertainties in the assessment of the condition of the structure at present and in the future. As an example, Bayesian decision theory, as presented in Raiffa and Schlaifer (1961), is applied to optimize the inspection efforts, see Yang and Trapp (1975), Madsen et al. (1989) and Faber et al. (2003). These approaches model the structure or the considered structural details as individual and homogeneous elements. However, structures, and in particular infrastructure, are large systems, which consist of many interdependent elements. Decisions regarding inspection and maintenance activities must be made by considering groups of elements, if not the entire structure, simultaneously, as demonstrated in Straub and Faber (2005). While classically the focus has been set on determining the optimal times and methods of inspections for a particular element, for structural systems, additionally, it is necessary to determine the percentage of the structure as well as the locations (elements) that should be inspected. Furthermore, inspection and monitoring results obtained from one element may be applied to update the deterioration models of the other elements in the structure.

In this paper, an overview is provided of the problems related to the planning and optimization of control and

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maintenance activities for large, deteriorating structural systems. This is followed by a proposal for a framework which facilitates the analysis of the spatial and temporal characteristics of the uncertain deterioration processes, as well as the optimization of the asset integrity management (AIM). In particular, emphasis is put on the representation of the spatial variability of the deterioration and its application in AIM. To this end, the classical life-cycle analysis and optimization, based on the pre-posterior analysis from the Bayesian decision theory, is extended from the consideration of individual elements to the analysis of (large) groups of interdependent elements. The proposed approaches are then illustrated by an application to a large reinforced concrete (RC) structure subjected to chloride-induced corrosion of the reinforcement.

2. Decisions in the asset integrity management of large, deteriorating structures

Decisions must be made during the entire life-cycle of a structure, i.e. during design, construction, operation, maintenance including inspection and monitoring, decommissioning and rebuilding. To optimize the total utility of the structure, in principle, these decisions have to be considered jointly, as they are all interrelated. As an example, the design and execution of a structure will strongly influence the deterioration performance in the later phases of its life-cycle. This paper concentrates on the decisions related to control and repair actions, assuming that the other decisions have already been taken, as is commonly the case for existing infrastructure.

Basically, the engineer responsible for the integrity of the structure must decide when, where and how to inspect or monitor and what to look for. Furthermore, the repair actions to be taken must be specified, based on the inspection results. In the following, Θ denotes the (uncertain) condition of the structure, e denotes the inspection and monitoring actions, Z denotes the inspection outcomes (capital letters indicate random variables) and d is the decision rule that specifies the repair actions as a function of the inspection outcomes. The corresponding optimization problem is then written as:

$$\min_{e,d} \mathbb{E}_{Z,e}[C_T(e,d,T_{SL})]$$

subject to

$$\Delta p_F(e,d,t) \leq \Delta p_F^{max},$$

$$t = 0, \ldots, T_{SL}. \quad (1)$$

$\mathbb{E}_{Z,e}[]$ denotes the expectation operation with respect to the uncertain parameters Θ and Z, $C_T$ is the total expected life-cycle cost, $T_{SL}$ is the anticipated service life time, $t$ is the time, $\Delta p_F$ is the annual probability of failure of the structure or the element and $\Delta p_F^{max}$ is the corresponding acceptance criterion. $\Delta p_F^{max}$ is prescribed by the owner or operator of the structure, in accordance with existing codes and regulations; however, for concrete structures most codes do not provide any acceptance criteria for deterioration limit states.

For individual elements, efficient strategies for solving the minimization problem in equation (1) have been developed (e.g. Straub and Faber (2006)) and have been successfully applied in practice (e.g. Faber et al. (2005)). At present, this is not the case for structural systems, mainly for two reasons. First, the modelling of the spatial characteristics of the deterioration has not been given sufficient attention in the past. Second, as discussed in Straub and Faber (2005), the set of possible inspection and repair strategies (e, d) increases exponentially with the size of the structure. The computation of equation (1), therefore, becomes intractable unless some restrictions on the minimization problem are introduced and a computationally efficient framework for the representation of the spatial characteristics of the deterioration exist. This paper addresses these problems by presenting such a framework for the case of large concrete surfaces subject to corrosion of the reinforcement.

3. Deterioration modelling for large systems

3.1 Temporal modelling

The stochastic modelling of deterioration processes has been extensively studied in the past. For RC structures subject to chloride-induced corrosion of the reinforcement, DuraCrete (1999) summarizes the state-of-the-art. Different limit state functions describe different so-called condition states, which are related to the serviceability and the safety of the structure. Typical condition states are illustrated in figure 1. The limit state functions describing corrosion initiation are relatively advanced for chloride and carbonation-induced corrosion and are based on physical (i.e. observable) parameters. Corrosion propagation is often modelled by simply defining the time period from

![Figure 1. Different condition states with time.](image-url)
initiation to visible corrosion, or to the occurrence of spalling. However, models are available which describe corrosion propagation (i.e. the events visible corrosion and spalling) by using physical parameters, e.g. Vu and Stewart (2002) or Val (2005). The limit state functions are then formulated in terms of the crack width.

The condition state of an element at time $t$ is denoted by $\Theta_t$. The probability of occurrence of the different condition states is described by the probability mass function (pmf) $p_\theta(\theta)$ and is calculated from the combination of the respective limit state functions (LSF). As an example, the probability of being in the condition state ‘corrosion initiation’ is assessed as the probability that corrosion has initiated and that visible corrosion has not (yet) occurred. When inspection or monitoring results $z$ are available, the probabilities of occurrence of the different condition states are updated accordingly, by applying limit state functions describing the inspection outcome, see Wicki et al. (2003).

The condition states are described in terms of limit state functions (LSF). The LSF represent the transition between two subsequent condition states. The LSF for corrosion initiation at time $t$ can be written as:

$$g_{CI}(t, x) = X_T \cdot T_I(x_0) - t,$$

where $X = [X_T, X_0]^T$ represents the vector of all random variables in the LSF. $T_I$ is the time till corrosion initiation as a function of the uncertain parameters $X_0$; the model uncertainty associated with $T_I$ is represented by $X_T$.

Considering chloride-induced corrosion, the ingress of chlorides through the concrete cover is described by a diffusion process with diffusion coefficient $D$ and it is assumed that corrosion will initiate when a certain critical concentration $C_{CR}$ is exceeded. Following DuraCrete (1999), the time till corrosion initiation at a depth of reinforcement $d$ is then written as:

$$T_I(x_0) = \frac{d^2}{4D} \left( \text{erf}^{-1} \left( 1 - \frac{C_{CR}}{C_S} \right) \right)^2,$$

where $C_S$ is the concentration of chlorides on the surface of the concrete. All parameters in equation (3) are here considered as random variables, i.e. $X_0 = [d, D, C_{CR}, C_S]^T$.

The LSF for visible corrosion can be written similarly as:

$$g_{VI}(t, x) = X_T \cdot T_I(x_0) + T_V - t,$$

with $T_V$ being the time from corrosion initiation to visible corrosion. For the example introduced later, physical modelling of $T_V$ is not considered, instead $T_V$ is modelled directly by a random variable. The vector of random variables in equation (4) is therefore $X = [X_T, X_0]^T, T_I]^T$.

3.2 Spatial modelling

For RC structures, the modelling of the spatial distribution of deterioration has received considerable attention in recent years, see Stewart and Faber (2003). The spatial characteristics of the processes causing initiation of corrosion at the reinforcement, such as chloride ingress and carbonation, have been modelled by a subdivision of the structure into individual elements, e.g. Hergenröder (1992), Faber and Rostam (2001) and Sterritt et al. (2001). The deterioration performance of these elements is, in general, interdependent, due to common influencing parameters, such as common materials, production processes and environmental influences.

The spatial variability is modelled by identifying zones of the structure with common properties. It is assumed that there is no interdependency between different zones. Different zones are, for example, the lower parts of the columns at a road that are subject to splash, and the upper parts of these columns. Thereby the lower parts of all columns are considered as one zone, i.e. interdependency between these parts at different columns is accounted for; the same holds for the upper parts.

Following the identification of homogeneous zones within a structure, the zones are further subdivided into elements. In order to assess the size of these elements, as well as their dependency structure, a study of the so-called correlation length or radius of the parameters governing the degradation process must be carried out. These parameters are modelled as random variables in the limit state function describing the corrosion process with time, see equation (2). The study of the correlation radius is based on a random field model, described in detail in, for example, Vanmarcke (1983) and Haldar and Mahadevan (2000) and on experimental data. The data can be either direct measurements of the parameters governing the degradation process or measurements of characteristics that experimentally have shown a good correlation with the governing parameters; the latter can then be estimated through regression analysis (Malioka et al. 2006). In both cases, the measurements should be performed such that both the small and large scale variability are taken into account, as described in Malioka and Faber (2004).

Consider, as an example of a structural component, the wall of a tunnel which is assumed to correspond to one single zone. If $u$ and $v$ denote the space coordinates and $X(u, v)$ denotes a concrete property along these space coordinates, then $X(u, v)$ can be represented by a random field with $X = X(u, v)$ being random variables and $f_X(x_1, x_2, \ldots, x_n)$ being the joint probability density function of the random variables. Assuming a homogeneous normal distributed random field of $n$ random variables (Li et al. 2003),
The multivariate normal density function is given by:

\[
f_X(x) = \frac{(2\pi)^{-n/2} |C_{XX}|^{-1/2}}{\sqrt{2}} \times \exp \left( -\frac{1}{2} (x - \mu_X)^T C_{XX}^{-1} (x - \mu_X) \right),
\]

where \( \mathbf{x} \) is the vector describing the material property under consideration at the different locations. \( C_{XX} \) is the \( n \) by \( n \) covariance matrix of the components in \( \mathbf{x} \), and \( \mu_X \) is a vector of size \( n \) containing the expected value of the random field. The covariance function of the random field is expressed as the product of the variance \( \sigma^2_{XX} \) and the correlation function \( \rho_{XX}(x_i, x_j) \). Some common choices for \( \rho_{XX}(x_i, x_j) \) are given in Madsen et al. (1986). As an example, assuming isotropic behaviour of the random field, the correlation function can be modelled by equation (6):

\[
\rho_{XX}(x_i, x_j) = \exp \left( -\frac{\sqrt{(u_i - u_j)^2 + (v_i - v_j)^2}}{r/\sqrt{2}} \right),
\]

where \( r \) is the correlation radius (or length). Based on the correlation radius, the zone considered can be divided into a number of elements within which strong correlation exists. The realizations of the material properties may then be approximated by independent random variables for the individual elements. An application of the approach can be found in Madsen et al. (1986).

3.3 A computational framework for the spatial deterioration characteristics based on hyper-parameters

As described in the previous section, structures are divided into homogeneous zones, which themselves are sub-divided into elements. The performance of the individual elements \( i \) is described by the condition states as a function of time, \( \Theta_{i,t} \), which the LSF describe. Without additional information, the elements in one zone are all identical, i.e. they are described by the same LSF and have identical deterministic and stochastic parameters.

However, it must be kept in mind that the determination of the material properties for a zone is, in general, subject to model uncertainties (statistical uncertainties due to a limited amount of data and uncertainties because the models are based on data from other structures). These model uncertainties have common realizations for all elements in the zone. For this reason, a dependency is introduced between

![Figure 2](image-url)
the parameters of the individual elements. Therefore, the individual $\Theta_{i,j}$ are also inter-dependent. It was first proposed to model these dependencies through a vector of the common influencing parameters by Faber et al. (2006). These parameters are termed hyper-parameters and are denoted by $A$. The hyper-parameters are time-invariant. In this framework, the parameters of the individual elements are a function of $A$ and the $\Theta_{i,j}$ are independent, given a particular realization of $A$. This model is depicted in figure 3.

The probability mass function (pmf) of the condition state of the element $i$ at time $t$ is thus:

$$p_{\Theta_{i,t}}(\theta_i) = \int_A p_{\Theta_{i,t}}(\theta_i|A)p_A(A)\,dA,$$  \hspace{1cm} (7)

where $f_A(A)$ is the joint probability density function of the hyper-parameters $A$. A similar framework is presented in Maes (2002), but is applied to the case where $f_A(A)$ is estimated for a particular structure from a set of inspection results (using a Maximum-Likelihood-Estimator), without specifying a prior distribution for $A$.

The determination of the hyper-parameters must follow from an analysis of the experimental evidence used for the determination of the random fields, which are the basis for the discretization of the surface in elements. As previously outlined, if the element size corresponds to the correlation length of the relevant material properties, the random variables describing the properties for the individual elements may be considered as conditional independent, given a particular realization of the hyper-parameters.

3.4 Computation of the spatial and temporal deterioration model

For computational convenience, in accordance with Maes (2002), it is proposed to discretize the joint distribution of the hyper-parameters $A$ and to replace the joint pdf $f_A(A)$ with the joint pmf $p_A(A)$. The integration in equation (7) is therefore replaced by a summation operation. The conditional pmf of the (discrete) condition states at the individual elements, $p_{\Theta_{i,t}}(\theta_i|A)$, can then be pre-calculated for all combinations of the discrete values of $A$ using simulation techniques or structural reliability analysis (SRA). This calculation is based on the limit state functions describing the condition states. The pre-calculated $p_{\Theta_{i,t}}(\theta_i|A)$ values are stored in a database, which is then utilized for the calculation of the probabilities for any specific element. With $p_A(A)$ and $p_{\Theta_{i,t}}(\theta_i|A)$, the state of the element $i$ at time $t$ is fully described given that no additional information is available. Furthermore, in this case all elements in one zone can be described by the same distribution function $p_{\Theta_{i,t}}(\theta_i|A)$. For a zone with $n$ elements, the probability of more than $x$ elements being in condition state $\theta^*$ at time $t$, can thus be assessed (see Faber et al. (2005)):

$$E_A[1 - B(x, n, p_{\Theta_{i,t}}(\theta = \theta^*|A))].$$ \hspace{1cm} (8)

$B(x, n, p)$ is the Binomial distribution function with argument $x$, sample size $n$ and probability $p$ and $E_A[\cdot]$ denotes the expectation operation with respect to the hyper-parameters.

When discretizing $A$, it must be ensured that the chosen discrete values for $A$ are sufficiently close. To this end, the discretization scheme is checked by comparing the reliability obtained with the discretized $A$ to that obtained from the model with the continuous distribution of $A$. As the emphasis is not in very small probabilities in the context of deteriorating concrete structures, the discretization of the tails of the distribution is generally not critical for this application.

3.4.1 Updating of the model with inspection results for individual elements. For most structures, information becomes available during the lifetime. For RC structures, different types of inspections are applied to assess the progress of deterioration with time (e.g. visual inspection, half-cell potential measurements, chloride measurement techniques) or to assess the initial quality of the construction (such as cover thickness measurements).

![Figure 3. Principle of the proposed spatial modelling.](image)
Inspection results for an element $i$ can, for any type of inspection, be described in the generic format $p_Z(z_i|\theta_i)$, with $z_i$ being the inspection result. Such inspection performance models have a long tradition, in particular for metallic structures with cracks and flaws. The so-called Probability of Detection (POD) model has been applied to this problem since the early 1970s, see Tang (1973). For concrete structures subject to corrosion of the reinforcement, inspection performance models have been developed for several non-destructive evaluation techniques (NDE) (see Wicki et al. 2003), which describe the probability of a specific inspection outcome given a particular condition of the inspected element or structure.

For individual elements, the updating of the model after an inspection at time $t_{\text{insp}}$ is straightforward by applying Bayes’ rule. For this purpose, a LSF for the inspection outcome must be defined from the inspection performance model, see Madsen et al. (1986). This LSF is $g_Z(t_{\text{insp}}, X)$. The probability of element $i$ being in condition state $\theta^*$ at time $t \geq t_{\text{insp}}$ is then obtained from:

$$p_{\theta^*}(\theta_i = \theta^*|z_i, \alpha) = \frac{P(g_{Z_i}(t, X) \leq 0 \cap g_Z(t_{\text{insp}}, X) \leq 0|\alpha)}{P(g_Z(t_{\text{insp}}, X) \leq 0|\alpha)}.$$  

(9)

Note that the formulation in equation (9) is conditional on $\alpha$, although the inspection result is assumed to be dependent only on the condition state of the element at the time of inspection:

$$p_{\theta_i}(z_i|\theta_i, \alpha) = p_{Z_i}(z_i|\theta_i).$$  

(10)

The updated probability of $\theta_i$ at time $t > t_{\text{insp}}$, however, is dependent on $\alpha$, due to the fact that the two LSF in equation (9) are evaluated at different times and are related only through the common random variables $X$. As $X$ is dependent on $\alpha$, the updated probability of being in a particular condition state is also dependent on $\alpha$. For the individual element, the updated $p_{\theta_i}(\theta_i|z_i)$ is then obtained in accordance with equation (7) by calculating the expected value of equation (9) with respect to $\alpha$.

Following equation (9), $p_{\theta_i}(\theta_i|z_i, \alpha)$ is calculated using simulation techniques or SRA. It is then stored in a database for all different values of $\alpha$ and all different potential inspection times and outcomes. To restrict the size of the database, only individual inspections are considered and not combinations of several inspections. This represents a simplification, as then only the most relevant inspection result for one element can be included in the analysis, and not the full history of inspection results.

### 3.4.2 Updating of the model with inspection results for entire zones

Inspections of one element will influence our estimate of the condition of all elements in the zone, due to the dependency in the deterioration performance as modelled by the hyper-parameters $\alpha$, see Faber and Sørensen (2002). In the following it will be demonstrated how this influence can be efficiently accounted for within the presented framework for modelling the spatial and temporal deterioration characteristics.

As shown in the previous section, the updating of the deterioration model for the individual elements is straightforward with the classical approaches. The updating of the deterioration model for the entire zone based on the inspection results from all elements, $Z = [Z_1, ..., Z_n]^T$, can be performed in a similar manner, by considering the LSF for all elements jointly. However, for larger zones such an approach is computationally very demanding and has thus limited value in practice. For this reason, a different procedure is proposed here, based on the database with the pre-calculated probabilities $p_{\theta_i}(\theta_i|\alpha)$, $p_{\theta_i}(\theta_i|z_i, \alpha)$ and $p_\alpha(\alpha)$.

A basic assumption of the framework is that the inspection result is independent of $\alpha$ given a particular state of the element at the time of inspection $t_{\text{insp}}$. By applying Bayes’ rule, we can therefore write:

$$p_\alpha(\alpha|t_{\text{insp}}, z_i) = p_\alpha(\alpha|t_{\text{insp}})$$

$$= \frac{p_{\alpha}(\alpha)}{\sum_\alpha p_{\alpha}(\alpha)p_\alpha(\alpha)}.$$  

(11)

where $p_\alpha(\alpha)$ is the prior distribution of the condition state in year $t_{\text{insp}}$ and is available from the pre-calculated database. However, the actual condition state of the element $i$ at $t_{\text{insp}}$ is unknown (otherwise no inspection would be required). To obtain the pmf of $\alpha$ given the inspection result $z_i$, $p_\alpha(\alpha|t_{\text{insp}})$ must be weighted with $p_{\theta_{\text{insp}}}(\theta_i|z_i)$:

$$p_\alpha(\alpha|z_i) = \text{E}_{\theta_{\text{insp}}}(p_\alpha(\theta_i|t_{\text{insp}}))$$

$$= \sum_\theta p_\alpha(\theta_i|t_{\text{insp}})p_{\theta_{\text{insp}}}(\theta_i|z_i).$$  

(12)

$p_{\theta_{\text{insp}}}(\theta_i|z_i)$ is obtained as:

$$p_{\theta_{\text{insp}}}(\theta_i|z_i) = \text{E}_\alpha[p_{\theta_{\text{insp}}}(\theta_i|z_i, \alpha)]$$

$$= \sum_\alpha p_{\theta_{\text{insp}}}(\theta_i|z_i, \alpha)p_\alpha(\alpha).$$  

(13)

As the inspection result of an individual element is independent of the results from the other elements given a particular value of the global parameters $\alpha$, the updating of the hyper-parameters can be performed
in a sequential manner. For example, for element 1 and 2 it is:

\[ p_A(x|z_1, z_2) = \frac{p_{Z_1}(z_2|x, z_1)p_A(x|z_1)}{\sum_x p_{Z_1}(z_2|x, z_1)p_A(x|z_1)} = \frac{p_{Z_1}(z_2|x)p_A(x|z_1)}{\sum_x p_{Z_1}(z_2|x)p_A(x|z_1)}. \] (14)

Note that equation (14) is not applicable for the case where several inspections of the same element are considered, due to the dependency between the inspection results.

The updated condition state of any element \( i \) given the last inspection results of all elements, \( Z = [z_1, \ldots, z_n]^T \), is then finally obtained as:

\[ p_{\Theta_i}(\theta_i|Z) = \sum_A p_{\Theta_i}(\theta_i|z_i, \alpha) \times p_A(x|z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_n). \] (15)

Together with the last NDE inspection, the results from the last visual inspection of the elements should be considered. Visual inspections are assumed to be perfect. After a visual inspection has been performed, it is known with certainty whether the inspected element is in the condition state ‘visible corrosion’ or not, for the year of the visual inspection, \( t_{VF} \). Given an indication, it is:

\[ P(\theta_{VF} = VC|z_{VF} = I) = 1, \] (16)

otherwise it is:

\[ P(\theta_{VF} = VC|z_{VF} = I) = 0, \] (17)

where \( VC \) stands for visible corrosion and \( I \) denotes an indication. For the time of the inspection it is thus:

\[ p_{\Theta_i}(z_{VF}|\theta_{VF}; z_{NDE}, \alpha) = p_{\Theta_i}(z_{VF}|\theta_{VF}), \] (18)

where \( z_{VF} \) denotes the result of the visual inspection and \( z_{NDE} \) denotes the result of the NDE inspection. The updated pmf of the condition state in the year of the visual inspection is thus:

\[ p_{\Theta_i}(\theta_i|z_{NDE}; z_{VF}, \alpha) = \frac{p_{\Theta_i}(\theta_i|z_{NDE}, \alpha)p_{\Theta_i}(z_{VF}|\theta_{VF})}{\sum_{\theta_i} p_{\Theta_i}(\theta_i|z_{NDE}, \alpha)p_{\Theta_i}(z_{VF}|\theta_{VF})}. \] (19)

For the years after the visual inspection, the probabilities can be updated in the same way, because if an element exhibits visible corrosion at time \( t_{VF} \), it will also do so at time \( t \geq t_{VF} \).

### 3.4.3 Calculating the updated probability of the number of elements in a condition state

For given values of the hyper-parameters \( \alpha \), the performance of an individual element in a zone is independent from the other elements. Therefore, for a given \( \alpha \) and when the number of elements is sufficiently large, the pdf of the number of elements \( n_0 \) in a particular condition state \( \theta \) can be approximated to a normal distribution \( f_{n_0}(n_0|\alpha, u) \). The moments of this normal distribution are obtained as:

\[ f_{n_0}(n_0|\alpha, u) \sim N(\mu_{n_0}, \sigma_{n_0}^2), \]

\[ \mu_{n_0} = \sum_{i=1}^n p_{\Theta_i}(\theta_i = \theta|z_i, \alpha), \]

\[ \sigma_{n_0}^2 = \sum_{i=1}^n \text{Var}[p_{\Theta_i}(\theta_i = \theta|z_i, \alpha)] = \sum_{i=1}^n p_{\Theta_i}(\theta_i = \theta|z_i, \alpha) - \left(p_{\Theta_i}(\theta_i = \theta|z_i, \alpha)\right)^2. \] (20)

The distribution of the number of elements in a particular condition state is now assessed by summation over the updated hyper-parameters:

\[ f_{n_0}(n_0|z) = \sum_A f_{n_0}(n_0|\alpha, u)p_A(\alpha|z). \] (21)

### 3.5 Definition and optimization of inspection and repair strategies

The criterion for the optimization of inspection and repair strategies is the expected total cost of the strategy, in accordance with equation (1). Different strategies, consisting of planned inspections and a decision rule on the repair actions based on the inspection outcomes, are considered. The expected cost of these strategies is calculated following the posterior analysis of the Bayesian decision theory.

Typical inspection strategies define the times of visual and NDE inspections as well as their extent (the inspection coverage). As all elements in a zone are identical before an inspection has been performed, it is not necessary to specifically identify the elements to be inspected. After the first inspections have been performed, it is beneficial to inspect the elements which have not already been inspected in past inspections. In this way it is ensured that the maximum information on the state of the entire zone is obtained (see also Straub and Faber (2005)).

The repair strategy is then given in terms of repair criteria, which specify the type of repair to perform as a function of the inspection outcome. As an example, it may be prescribed that minor repairs of all elements in the zone (corresponding to a treatment of the concrete cover) are performed when the number of elements with an indication
of initiated corrosion at an inspection exceeds $\gamma\%$. Such a criterion has to be defined with separate values for different inspection types. Additionally, it would be prescribed that major repairs (including a replacement of the reinforcement) are performed when $\delta\%$ of the surface of the zone exhibits visual corrosion. This criterion corresponds to a failure criterion, i.e. $\delta\%$ of the surface exhibiting visual corrosion is the undesirable and costly event whose probability is reduced by previously performed inspections and minor repairs. The optimal inspection and repair strategy thus represents a trade-off between the cost of inspections and minor repairs and the cost of major repairs.

Here, it is assumed that the event of visual corrosion of more than $\delta\%$ of the surface is always detected and repaired in due time and that the structure does not fail because of corrosion of the reinforcement. This is clearly a simplified model and neglects potential structural failures caused by the corrosion of the reinforcement, such as spalling or collapse of the structure. However, if the criterion for repair, $\delta$, is sufficiently low, as is the case in many European countries, the probability of such failures are low and the expected cost of repairs are by far larger than the expected cost of potential structural failures. Otherwise, the failure criterion as well as the cost of a failure must be adjusted to account for the effective maintenance policy and must eventually include the expected cost of structural failure.

To calculate the expected costs for a given strategy, the probabilities of all different possible inspection outcomes are determined. With given decision rules regarding the repair actions to perform as a function of the inspection outcomes, it is then possible to calculate the probability of the different possible performances of the structure. The expected cost of an inspection and repair strategy is then obtained by an expectation operation over the different inspection outcomes and performances of the structure.

In figure 4, the corresponding decision tree is illustrated. This model is based on the assumption that repair or failure only apply to the entire zone or structure and that a repaired or failed zone performs as a new one. It is reminded that the event of failure actually corresponds to a major repair of the zone.

As discussed in Straub and Faber (2005), for systems with many elements, the number of different combinations of inspection outcomes becomes very large. This makes the computation of the expected cost a time-consuming task. The efficient algorithm to calculate the probability of the state of the system (i.e. the probabilities of the different branches in the decision tree), as presented in the previous sections, is therefore crucial for this task. Still, it is in general necessary to restrict the decision tree, and the evaluation of the tree. As an example it is possible to neglect all branches in the tree with an occurrence probability of less than $\varepsilon_{\text{lim}}$. The error made by such a
truncation of the decision tree can be appraised by varying $\varepsilon_{\text{lim}}$. A typical value of $\varepsilon_{\text{lim}}$ is $10^{-5}$.

4. Example

For the purpose of illustration, the framework is applied to an example structure. The temporal and spatial deterioration model is adopted from Faber and Sorensen (2002). The different condition states are described by the limit state functions in equations (2) and (4). The parameters, which are identical for all elements in the zone, are provided in table 1.

Due to statistical uncertainty, the mean value of the diffusion coefficient $D$ and the surface concentration $C_S$ are assumed to be uncertain themselves. The realizations of $\mu_D$ and $\mu_{C_S}$ are the same for the entire zone. These mean values are thus hyper-parameters $A$ of the model and cause a stochastic dependency between the performances of the individual elements in the zone. Each of the hyper-parameters is represented by 11 discrete points, $A$ is thus given by $11 \times 11 = 121$ states. The discrete values of $\mu_D$ (in mm$^3$/yr) are:

$27.6, 31.2, 34.0, 36.0, 38.0, 40.0, 42.0, 44.0, 46.0, 48.8, 52.4$;

the values of $\mu_{C_S}$ (in wt.% of concrete) are:

$0.276, 0.312, 0.34, 0.36, 0.38, 0.4, 0.42, 0.44, 0.46, 0.488, 0.524$.

The chosen discretization points are symmetrical around the mean. This choice is based on the fact that both $\mu_D$ and $\mu_{C_S}$ are Normally distributed and that the probabilities of interest are in the range of 0.05 to 0.95. The choice of the discretization points is checked by comparing the obtained (unconditioned) reliability to the one calculated with the continuous distributions for $\mu_D$ and $\mu_{C_S}$. As the resulting reliabilities match closely in the area of interest, the choice is deemed appropriate.

Half-cell potential measurements (HCPM) are considered. Following Faber and Sorensen (2002), the applied model for the inspection quality, corresponding to equation (10), is:

\[P(z_i = I|\theta = CI) = 0.9 \quad \text{and} \quad P(z_i = I|\theta = \overline{CI}) = 0.29,\]

with $I$ indicating an initiation and $CI$ denoting the state of initiated corrosion or any following state.

In addition, visual inspections of the surface can be performed. Following equations (16) and (17) it is assumed that these inspections are perfect and, therefore:

\[P(z_{\text{VI}} = I|\theta = VCI) = 1 \quad \text{and} \quad P(z_{\text{VI}} = I|\theta = \overline{VCI}) = 0.\]

The probability of being in a particular condition state is now pre-calculated for different values of the hyper-parameters and different inspection times and outcomes using the Monte Carlo simulation. For a particular element, the probabilities of being in any of the different condition states is then assessed using equation (7). The result is shown in figure 5.

In the following, a zone with 100 identical elements is considered over a service life period of 50 years. Applying the framework outlined in this paper, any type of inspection outcome can now be considered. An example of inspection outcomes and the respective probabilities of having visible corrosion in year 50 are shown in figure 6. Figure 6(a) presents the inspection results and inspection times of the 18 elements which have been inspected using HCPM. Figure 6(b) presents the resulting probability of being in the state of visible corrosion for all elements. Note that, with these inspection outcomes, for the non-inspected elements the probability of having visible corrosion decreases to 0.35 from an original 0.43 according to figure 5.

In figure 7, the same case is considered, but additionally a visual inspection of the upper part of the zone is performed in year 40. It is observed that this inspection changes the probabilities significantly.

It must be remembered that the states of the individual elements are not independent. Therefore, the number of elements in a particular condition state must be evaluated following equation (21). Figure 8 shows the results of such an assessment for two different inspection outcomes. Note that the probability of having no element with corrosion initiated is 0.004 for the prior model (without inspections) and 0.016 for the case where all inspections result in no-indication. This is due to the dependency in the element performances.

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-normal</td>
<td>55.0</td>
<td>$\mu_D$</td>
<td>Log-normal</td>
<td>Log-normal</td>
<td>Log-normal</td>
<td>Log-normal</td>
<td>Normal</td>
<td>Normal</td>
</tr>
<tr>
<td>Mean value</td>
<td>11.0</td>
<td>10.0</td>
<td>0.08</td>
<td>0.05</td>
<td>0.05</td>
<td>1.88</td>
<td>4.0</td>
<td>0.04</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</table>
Due to the efficiency of the presented computational framework, the calculation of the above results takes less than one CPU-second on a standard PC, once the database with the probabilities for given values of the hyper-parameters is available. The framework is therefore suited for the demanding calculation of the expected cost of an inspection/repair strategy, in accordance with figure 4.

In the following, an example considering the optimization of HCPM inspection time and coverage as well as the repair strategy in a zone with 100 elements is presented. The decision maker has determined that a major repair must be performed when more than 30% of the zone exhibits visible corrosion. The repair strategy is that a minor repair of the zone is performed if more than \( \chi \)% of

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**Figure 5.** Probabilities of being in any adverse condition state as a function of time for an element before any inspection.

**Figure 6.** Probabilities of visible corrosion for the entire zone for HCPM at years 25 and 35.
the HCPM result in an indication, \( \chi \) is an optimization parameter. The cost model is presented in Table 2.

In Figure 9, the temporal distribution of the expected failure cost (the cost of a major repair) is presented for the case where no inspections (and minor repairs) are performed, with a corresponding total expected cost.

Figure 7. Probabilities of visible corrosion for the entire zone after HCPM at years 25 and 35 (according to Figure 6) and a visual inspection at year 40.

Figure 8. Probability mass function of the number of elements with initiated corrosion in year 50, considering different inspection outcomes at year 25.

Table 2. Parameters of the cost model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of inspection an element using HCPM</td>
<td>10</td>
</tr>
<tr>
<td>Cost of minor repair per element</td>
<td>50</td>
</tr>
<tr>
<td>Cost of major repair per element</td>
<td>200</td>
</tr>
<tr>
<td>Interest rate (yr(^{-1}))</td>
<td>0.02</td>
</tr>
<tr>
<td>Service life (yr)</td>
<td>40</td>
</tr>
</tbody>
</table>
(present value) of 3687. This is compared with a strategy including a HCPM at year 30, resulting in a total expected cost of 2452.

As observed from figure 9(a), the probability, and therefore the expected cost, of a major repair increase drastically after year 25. It follows, that an inspection is most effective in this period of the service life; this is also confirmed by a numerical investigation, resulting in an optimal time for the HCPM inspections at year 30.

Apart from the inspection time, the repair criterion $\gamma$ is varied. This is presented in figure 10. The expected failure cost increases with increasing $\gamma$, whereas the expected cost of minor repairs decreases.

In figure 11, the expected costs are presented as a function of the inspection coverage. It is observed that the optimal inspection coverage is in fact 100%. This result points to the fact that the dependency between the performances of the individual elements is relatively weak with the applied model. However, the optimal coverage also depends on the ratio between inspection and repair costs and may thus decrease when higher inspection costs occur.

Figure 9. Temporal distribution of expected costs (not discounted): (a) without any inspection and repairs, and (b) HCPM of 50% of the elements in year 30, with a repair criterion of 45% indication.
Figure 10. Expected costs for a strategy with HCPM of 50% of the elements in year 30 as a function of the repair criterion (a minor repair is performed if more than \(w\)% of the inspections result in an indication).

Figure 11. Expected costs for a strategy with HCPM in year 30 as a function of the inspection coverage, when a minor repair is performed if more than 45% of the inspections result in an indication.
5. Concluding remarks

The framework presented here enhances the risk based planning and optimization of inspection efforts for large deteriorating concrete structures. The core of the presented framework is a computationally efficient algorithm for assessing the joint probability of the condition states of the elements, which are described by means of limit state functions, including the effect of performed inspections. This algorithm is based on a pre-calculation of the conditional probabilities for the individual elements (conditional on particular values of the hyper-parameters and particular inspection outcomes).

A main feature of this approach is the discretization of the hyper-parameters \( A \). For the example presented in this paper, discretization of the hyper-parameters \( A \) is straightforward and computationally very efficient. However, the required computation efforts are proportional to the number of states in \( A \). In the example, two random variables were considered as hyper-parameters and each one was represented by 11 discrete values, leading to a total of \( 11 \times 11 = 121 \) states in \( A \). If \( n \) additional hyper-parameters were included in the analysis in a similar manner, the computational time of the calculation is increased approximately by a factor of \( 11^n \); the size of the database with the conditional probabilities increases accordingly. Therefore, the crude discretization scheme applied here must be refined if more than 4 to 5 hyper-parameters are included in the analysis. However, it is believed that in most instances this situation can be avoided with little loss of accuracy by including only the most influential hyper-parameters in the analysis.

For many structures, several types of potential deterioration mechanisms exist, and these must be considered jointly in the optimization, in particular when the same inspection methods may be applied for the different mechanisms. While the presented example considers chloride-induced corrosion of the reinforcement, the computational framework is equally applicable to other types of deterioration, when LSF for describing the different condition states are available. Such models are for example published for corrosion of the reinforcement caused by carbonation of the concrete. As some of the random variables are the same in the individual LSF (e.g. the cover thickness \( d \)), the different deterioration mechanisms should be analysed jointly by defining the condition states through a combination of the LSF (e.g. the condition state corrosion initiation would be defined as the union of the events corrosion initiation due to carbonation and corrosion initiation due to chloride ingress).

Finally, it is concluded that the framework presented has a large potential for an application on concrete structures with large areas with similar conditions such as tunnels, bridges and retaining walls. For such structures, the efforts required for quantitative modelling of the deterioration will be far outweighed by the savings on repair and inspection costs.

References


