Designing for wind actions based on time-domain analysis: Accounting for statistical uncertainty

Daniel Straub, Iason Papaioannou
Engineering Risk Analysis Group, Technische Universität München
straub@tum.de, iason.papaioannou@tum.de

Alexander Michalski
SL-Rasch GmbH, Stuttgart, alex.michalski@sl-rasch.de

ABSTRACT: Engineers designing structures must work with incomplete and imperfect models. In standard design situations, safety provisions in codes implicitly account for these uncertainties. However, for more advanced design procedures that are not covered by the code, e.g. when dealing with non-linear dynamic problems, the engineer must explicitly address this uncertainty. A special case of uncertainty arises when time domain analysis is applied for determining the extreme response under wind loading: The statistical uncertainty due to a limited duration of the dynamic simulation. In this paper, we discuss this uncertainty and propose a reliability-based approach to account for this uncertainty in a semi-probabilistic design format. A quantitative relation between the computational efforts made in design and the additional safety required is established. Numerical investigations are performed for large membrane structures analyzed by means of Fluide Structure Interaction (FSI) simulation.

1 INTRODUCTION

Engineers designing structures must work with incomplete and imperfect models (Ditlevsen and Madsen 1996; Der Kiureghian and Ditlevsen 2009). In standard design situations, safety provisions in codes (e.g. Eurocode 0) implicitly account for these uncertainties. However, for more advanced design procedures that are not covered by the code, e.g. when dealing with non-linear dynamic problems, the engineer must explicitly address this uncertainty. A special case of model uncertainty arises when time domain analysis is applied for determining the extreme response under wind loading: The statistical uncertainty due to a limited duration of the dynamic simulation. As will be shown in this paper, this uncertainty can be considerable for realistic design situations.

In Eurocode 0, reference to this type of uncertainty is made in paragraph 5.2, which deals with design assisted by testing: “The statistical uncertainty due to a limited number of test results shall be taken into account.” The informative annex D of Eurocode 0 provides some additional guidance on how to deal with the uncertainty, but specific recommendations are made only for the case of resistance variables that have the Normal or Lognormal distribution. (The recommendations are based on, among others, work by Rackwitz (1983).) No detailed guidance is provided for the case where design values for load actions or load effects are determined based on limited samples, e.g. from a time series that is obtained either by numerical simulation or experiments (e.g. wind tunnel testing). To propose a criterion and a procedure for dealing with the statistical uncertainty in these cases is the goal of the present paper.

We discuss the uncertainty and propose a reliability-based approach to account for it in a semi-probabilistic design format. A quantitative relation between the computational efforts made in design and the additional safety required is established. Numerical investigations are performed for large membrane structures analyzed by means of Fluide Structure Interaction (FSI) simulation (Michalski et al. 2011).

2 DETERMINING THE EXTREME RESPONSE OF MEMBRANE STRUCTURES SUBJECT TO WIND LOADS

2.1 *Structural analysis*

The reliability of membrane structures (e.g. Figure 1) is commonly determined by extreme wind loads due to their large surface-to-mass ratio and their flexibility. Particularly transient wind load combined with wide spans and low pre-stress levels of the membrane can lead to dynamic amplifications of the structural response. The assessment of dynamic response of membrane structures is highly complex due to their special load carrying behavior, their material properties and their distinct structural interaction with flow induced effects.

Figure 1. Medina Piazza Shading Project, 26m Umbrellas, Medina, Saudi Arabia, SL-Rasch GmbH.

Common methods in wind engineering practice, such as small scale wind tunnel experiments, do not fully cover non-linear structural behavior, contact interaction between membrane and structural elements and the interaction of the flow field with the structural response. Therefore, numerical tools – developed and validated during several scientific and applied engineering studies (Michalski 2010, Michalski et al. 2011) – are proposed to overcome limitations of existing structural analysis approaches and are used for structural engineering.

2.2 *Computational wind engineering methodology*

The complete simulation methodology, consisting of the numerical wind flow simulation and the fluid–structure coupling simulation, is summarized in Fig. 2. With this simulation approach it is possible to examine all aspects of wind-loaded membrane structures. The applied fluid structure interaction (FSI) simulation methodology allows the realistic description of the nonlinear structural behavior at the real scale, which is especially important in the case of textile structures, and of the stochastic wind excitation. The analysis is performed in time-domain.

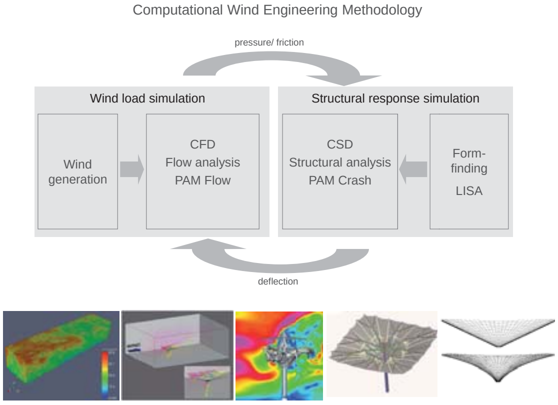

Figure 2. Computational Wind Engineering Methodology using a fully partitioned FSI approach.

The unknown parameters of the flow (velocity and pressure) as well as the structure (forces and deformations) are calculated including the fluid–structure coupling conditions. It should be noted that for wind load assessments by applying CFD techniques, an accurate time dependent analysis is required. Therefore turbulent flow is modeled by large eddy simulation (LES) based on the Smagorinsky subgrid scale (SGS) model. It is used because it enables the prediction of peak pressures and maximum/minimum structural response in the FSI context.

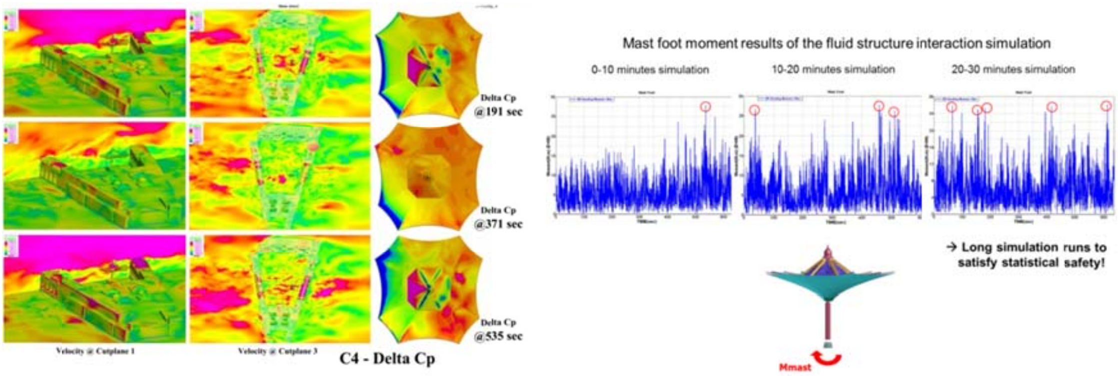

Figure 3. Time domain FSI analysis of large umbrella structures (left: CFD/LES pressure contour results, right: structural response results)

2.3 *Design requirement*

The load-and-resistance-factor-design (LRFD) principle implemented in Eurocode 0 requires that structural elements comply with the following design requirement:

$$\frac{R_k}{\gamma_R} \geq E_d \tag{1}$$

Wherein:

R_k: characteristic value of the capacity R;
γ_R: partial safety factor for the capacity R; and
E_d: design value of the load effect.

In the standard Eurocode approach, the design load effect E_d is determined as a function of the design wind action on the structure $Q_d = Q_k \gamma_Q$, with Q_k being the characteristic wind action and γ_Q the partial safety factor for wind loads. However, due to the coupling of the actions to the structural response (deformations), such an approach is not meaningful for the considered structures. Instead, the non-linear coupled structural analysis is performed for a characteristic wind field $\mathbf{V}_k$ and characteristic weight G_k, resulting in a characteristic load effect E_k. The design load effect is then determined as

$$E_d = E_k \gamma_Q \tag{2}$$

The characteristic wind field $\mathbf{V}_k$ is defined based on the 50 year wind speed (corresponding to a 98% quantile in of the annual maximum wind speed). The characteristic load effect E_k is defined as the expected value of the maximum response during a 10 min time period in which the structure is subjected to $\mathbf{V}_k$.

Note that for the considered application (Sec. 2.1) this approach is on the conservative side when compared to the standard Eurocode procedure of determining the response as a function of the design action Q_d. In the static or quasi-static analysis of Eurocode 0, the characteristic action Q_k acting on the structure is proportional to the square of the wind velocity v^2. If the load effect was a linear function of v^2, it would be irrelevant whether the safety factor γ_Q were introduced at the level of the load action or the level of the load effect. For the considered application, it was found by numerical checks that the load effects increase under-proportional to v^2. Introducing the safety factor γ_Q at the level of the load effect thus leads to larger design loads and is conservative.

2.4 *Extreme value analysis to determine the characteristic load effect*

For a given characteristic wind field $\mathbf{V}_k$, the FSI analysis results in a time series of the relevant load effect with length T, an example of which is provided in Fig. 2. This time series $x(t)$ represents one realization of the stochastic process $X(t)$ of the load effect resulting from the considered wind field $\mathbf{V}_k$.

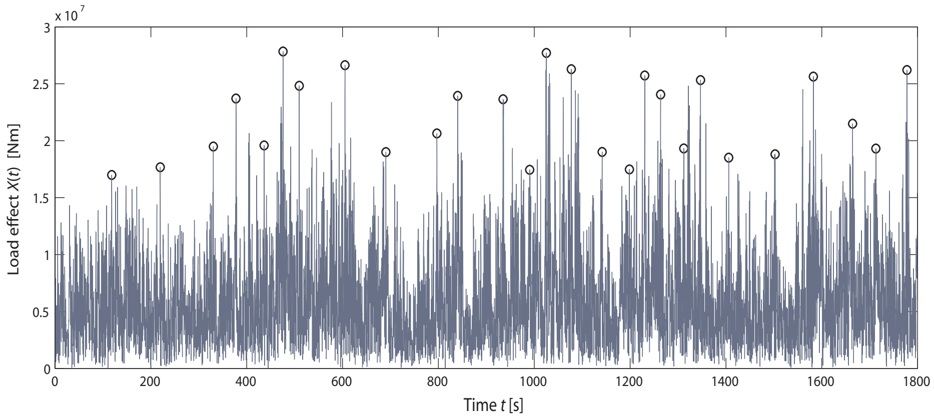

Figure 4. Time series of a load effect (resulting moment) obtained from the FSI analysis, with peaks identified using the declustering algorithm of Tawn (1988).

Through a series of tests, it is found that $X(t)$ can be considered a stationary process. In addition, it is assumed to be ergodic and have limited long-range dependence at extremal levels.

Let Y denote the maximum of $X(t)$ during a 10 min period:

$$Y = \max X(0{:}\,10\text{min}) \tag{3}$$

The distribution of Y can be estimated from a time series $x(t)$ as in Fig. 2 using extreme value theory (Coles 2007). Both the Peak-over-threshold (POT) and the block-maxima approach are implemented and results are compared. For brevity, only the latter is reported here. With the block-maxima approach, the data $x(t)$ is separated in blocks of length b. A value $b = 60$ s is used. The set of maximum observed values in each block $x_{m,1}, \dots, x_{m,n}$ is identified and a Generalized Extreme Value (GEV) distribution is fitted to this data using a Maximum Likelihood Estimator (MLE). The GEV distribution is:

$$F_{X_m}(x;\boldsymbol{\theta}) = \exp\left\{-\left[1+\frac{\beta(x-\epsilon)}{\alpha}\right]^{-\frac{1}{\beta}}\right\}, \qquad x \geq \epsilon - \frac{\alpha}{\beta} \tag{4}$$

where $\boldsymbol{\theta} = [\alpha;\beta;\epsilon]$ are the parameters of the distribution (scale, shape and location parameter, respectively) . This is the distribution of X_m, the maxima in each block of duration b. The distribution of Y, the maxima in 10 min, is

$$\begin{aligned} F_Y(y;\boldsymbol{\theta}) &= \left[F_{X_m}(y;\boldsymbol{\theta})\right]^{\left(\frac{10\text{min}}{b}\right)} \\ &= \exp\left\{-\left(\frac{10\text{min}}{b}\right)\left[1+\frac{\beta(y-\epsilon)}{\alpha}\right]^{-\frac{1}{\beta}}\right\} \\ &= \exp\left\{-\left[1+\frac{\beta\left(y-\left(\epsilon-\frac{\alpha}{\beta}+\frac{\alpha}{\beta}\left(\frac{10\text{min}}{b}\right)^{\beta}\right)\right)}{\alpha\left(\frac{10\text{min}}{b}\right)^{\beta}}\right]^{-\frac{1}{\beta}}\right\}, \qquad y \geq \epsilon - \frac{\alpha}{\beta}. \end{aligned} \tag{5}$$

This corresponds to a GEV distribution with parameters $\alpha_{10} = \alpha\left(\frac{10\text{min}}{b}\right)^{\beta}$, $\beta_{10} = \beta$ and $\epsilon_{10} = \epsilon - \frac{\alpha}{\beta} + \frac{\alpha}{\beta}\left(\frac{10\text{min}}{b}\right)^{\beta}$.

For given parameter values $\boldsymbol{\theta}$, the characteristic value of the load effect E_k is defined as the expected value of the maximum of $X(t)$ in a 10 min period, i.e. the expected value of Y:

$$\begin{aligned} E_k(\boldsymbol{\theta}) &= \mathrm{E}(Y|\boldsymbol{\theta}) \\ &= \int_{\epsilon-\frac{\alpha}{\beta}}^{\infty} [1 - F_Y(y;\boldsymbol{\theta})]\mathrm{d}y \\ &= \epsilon_{10} + \frac{\alpha_{10}}{\beta_{10}}[\Gamma(1-\beta_b) - 1] \\ &= \epsilon + \frac{\alpha}{\beta}\left[\left(\frac{10\text{min}}{b}\right)^{\beta}\Gamma(1-\beta) - 1\right]. \end{aligned} \tag{6}$$

The third identity follows from the fact that E_k has the GEV distribution with parameters α_{10}, β_{10} and ϵ_{10}.

The MLE of $\boldsymbol{\theta}$ is computed from the observed block maxima $x_{m,1}, \dots, x_{m,n}$ by:

$$\boldsymbol{\theta}_{\text{MLE}} = \max_{\boldsymbol{\theta}} l(\boldsymbol{\theta}) = \max_{\boldsymbol{\theta}} \sum_{i=1}^{n} \ln f_{X_m}(x_i;\boldsymbol{\theta}) \tag{7}$$

Figure 3a shows the empirical and the fitted probability distribution of X_m for the extremes of the time series shown in Fig. 2. The probability distribution of Y with the fitted parameters $\boldsymbol{\theta}_{\text{MLE}}$ is shown in Fig. 3b.

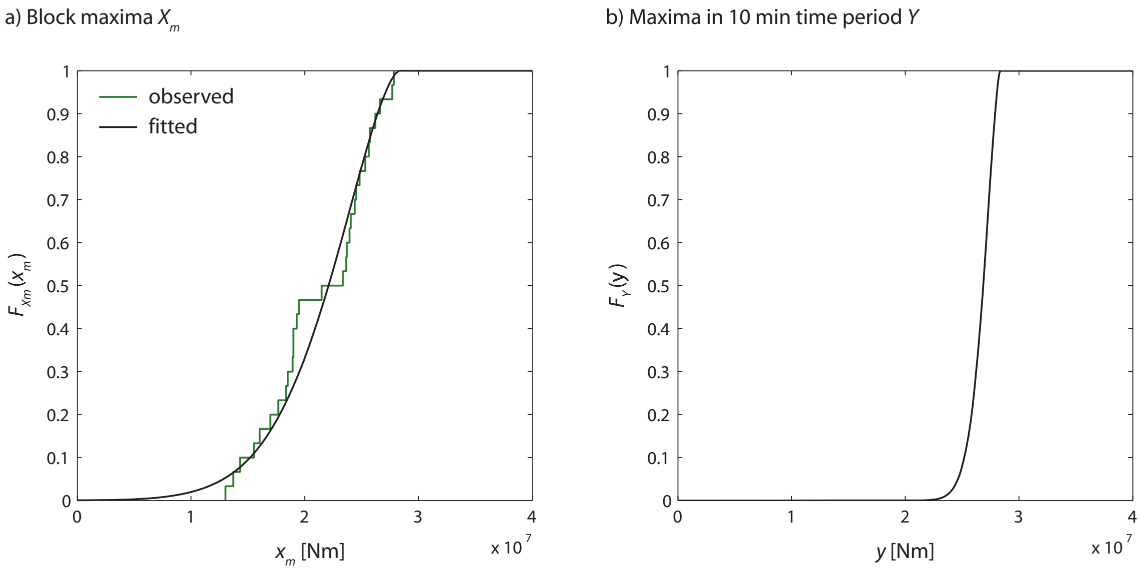

Figure 5. (a) Empirical and fitted probability distribution of block maxima X_m obtained from the time series in Fig. 4; (b) corresponding probability distribution of Y, the maximum load effect in a 10 min period.

2.5 *Statistical uncertainty*

A Bayesian estimate of $\boldsymbol{\theta}$ is applied for representing the statistical uncertainty associated with the limited sample size. We use the asymptotic Normal approximation of the likelihood. Using a non-information prior distribution, this leads to a multivariate Normal posterior distribution of $\boldsymbol{\theta}$ given the data $x_{m,1}, \dots, x_{m,n}$:

$$\boldsymbol{\theta}|x_{m,1}, \dots, x_{m,n} \sim \mathrm{MVN}(\boldsymbol{\theta}_{\mathrm{MLE}}, \mathbf{C}_{\boldsymbol{\theta\theta}}). \tag{8}$$

where the covariance matrix $\mathbf{C}_{\boldsymbol{\theta\theta}}$ is the inverse of the observed Fisher information matrix $\mathbf{I}_{\mathrm{O}}$ evaluated at the MLE (Coles 2007):

$$\mathbf{C}_{\boldsymbol{\theta\theta}} = \mathbf{I}_{\mathrm{O}}(\boldsymbol{\theta}_{\mathrm{MLE}})^{-\mathbf{1}} \tag{9}$$

The observed Fisher information matrix $\mathbf{I}_{\mathrm{O}}(\boldsymbol{\theta})$ is equal to the Hessian of the log-likelihood $l(\boldsymbol{\theta})$ with respect to the parameters $\boldsymbol{\theta}$.

For the limiting case of an infinite time series, $\mathbf{C}_{\boldsymbol{\theta\theta}}$ is zero and $\boldsymbol{\theta}$ is deterministically equal to $\boldsymbol{\theta}_{\mathrm{MLE}}$.

Based on the posterior distribution of $\boldsymbol{\theta}$, the posterior probability density function of the characteristic value E_k, f''_{E_k}, can be determined. Due to the non-linearity of the function $E_k(\boldsymbol{\theta})$, Eq. (6), the distribution f''_{E_k} can only be determined numerically. Here, Monte Carlo Simulation (MCS) is used for this purpose. Alternatively, numerical integration or a first-order approximation can be applied. The first-order approximation results in a Normal distribution with mean and variance as follows:

$$\hat{\mu}_{E_k} = E_k(\boldsymbol{\theta}_{\mathrm{MLE}}), \tag{10}$$

$$\hat{\sigma}^2_{E_k} = \nabla E_k^T \mathbf{C}_{\boldsymbol{\theta\theta}} \nabla E_k, \tag{11}$$

with

$$\nabla E_k = \left[\frac{\partial E_k(\boldsymbol{\theta})}{\partial \alpha}, \frac{\partial E_k(\boldsymbol{\theta})}{\partial \beta}, \frac{\partial E_k(\boldsymbol{\theta})}{\partial \epsilon}\right]^T, \tag{12}$$

evaluated at $\boldsymbol{\theta} = \boldsymbol{\theta}_{\mathrm{MLE}}$.

For illustrative purposes, in Fig. 4 the resulting posterior distribution of E_k is shown for different lengths T of the simulated time series $x(t)$. This distribution is obtained with the data of Fig. 2. This illustrates the effect of T on the statistical uncertainty in the design parameter. As expected, the posterior variance of E_k is decreasing with increasing length of the time series. Furthermore, it is observed that the first-order approximation is underestimating the true variability of E_k, in particular for shorter time series.

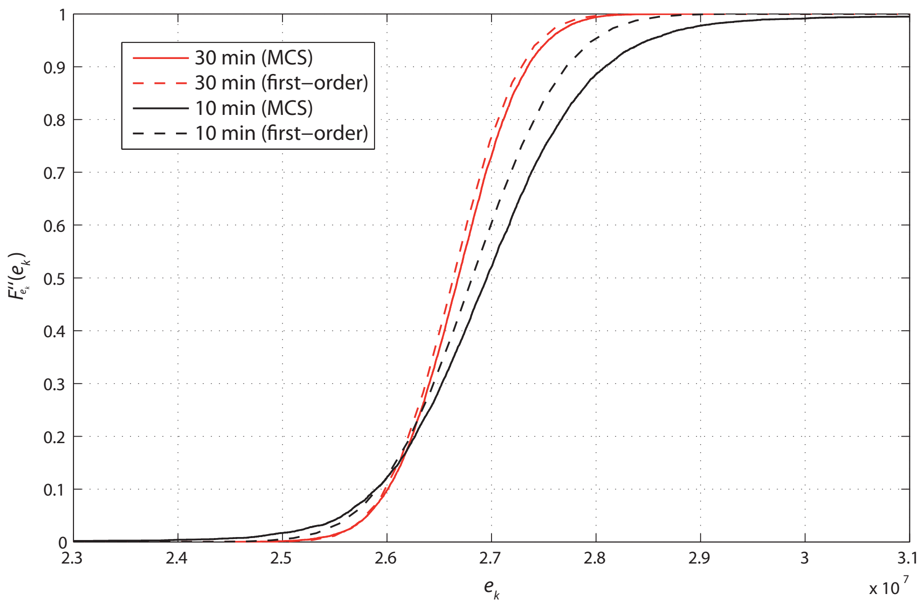

Figure 6. Example of F''_{E_k}, the posterior distribution of the characteristic load effect E_k, for different lengths T of the simulated time series $X(t)$. Results as obtained with MCS and with a first-order approximation are shown.

What is the interpretation of f''_{E_k}, and what is its relevance in the design? For the given wind scenario, there is one true value of E_k, which – under the assumption of ergodicity – could be determined if a time series $x(t)$ of infinite duration $T = \infty$ were available. In practice, due to a limited duration T, it is only possible to determine a distribution of E_k as shown above. If the MLE estimate $E_k(\boldsymbol{\theta}_{\text{MLE}})$ were employed, there is a large probability (in the order of 50%) that the true value of E_k is underestimated, leading to a non-conservative design. For this reason, instead of $E_k(\boldsymbol{\theta}_{\text{MLE}})$ an upper quantile value of the distribution f''_{E_k} should be used in design. Let E_{kq} denote this upper quantile. But which is the appropriate quantile level q to be applied? This question will be addressed in the next section.

3 RELIABILITY-BASED DETERMINATION OF QUANTILES FOR DESIGN

3.1 *Quantile value of E_k*

To account for the statistical uncertainty described in Section 2.5, upper quantile values of f''_{E_k} must be used in the design. However, no guidance on the appropriate quantile level q is found in Eurocode or other literature. Therefore, the following criterion is proposed to determine the appropriate quantile level:

The level of the upper quantile is selected so that the reliability of a design based on a limited time series $x(t)$ of duration T is equal to the reliability of a design based on an infinite time series (when no statistical uncertainty is present).

Because structural analyses are only performed for a characteristic wind field $\mathbf{V}_k$, it is not possible to actually compute the reliability. Instead, the reliability conditional on the characteristic wind field $\mathbf{V}_k$ is computed and the above criterion is applied to this conditional reliability. (Note that we make no assessment of the appropriateness of the characteristic wind field.) Let β_k denote this conditional reliability index. To make explicit the dependence on the computation, let $\beta_k^{(T,q)}$ denote the conditional reliability index obtained from a design based on a limited time series of length T and using an upper quantile level q. The goal is thus to find a value of q that fulfills the following condition:

$$\beta_k^{(T,q)} = \beta_k^{(\infty)} \tag{13}$$

$\beta_k^{(\infty)}$ is the conditional reliability index obtained from a design based on infinite time series. In the following, the probabilistic model and the computation of $\beta_k^{(T,q)}$ and $\beta_k^{(\infty)}$ are presented.

3.2 *Probabilistic model*

E is the maximum value of the load effect occurring during the 10min duration of the representative wind scenario. It is modeled as:

$$E = Y \cdot Z_m, \tag{14}$$

with Z_m being the model error of the structural (FSI) analysis. The distribution of Z_m is estimated from previous experimental validations of the FSI, as reported in Michalski et al. (2011), as the Lognormal distribution with mean value 1 and standard deviation 0.25. To assess the sensitivity of the final results with respect to this parameter, alternative choices were investigated, which are not reported here.

For given parameters $\boldsymbol{\theta}$, Y has the probability distribution $F_Y(y; \boldsymbol{\theta})$ according to Eq. (5). Because $\boldsymbol{\theta}$ itself is a random variable with posterior distribution $f''_{\boldsymbol{\theta}}(\boldsymbol{\theta})$ according to Eq. (8), Y is described by its predictive distribution, defined as

$$\tilde{F}_Y(y) = \int_{\boldsymbol{\theta}} F_Y(y; \boldsymbol{\theta}) f''_{\boldsymbol{\theta}}(\boldsymbol{\theta}) \mathrm{d}\boldsymbol{\theta}. \tag{15}$$

This integral is evaluated numerically by means of MCS. For the reference case (with $T = \infty$), $\boldsymbol{\theta} = \boldsymbol{\theta}_{\mathrm{MLE}}$ deterministically, and the predictive distribution reduces to

$$\tilde{F}_Y(y) = F_Y(y; \boldsymbol{\theta}_{\mathrm{MLE}}). \tag{16}$$

The capacity R is modeled by a Lognormal distribution with coefficient of variation 0.1. The mean value of R is determined through the design criterion as follows.

3.3 *Design criterion*

The design criterion is given by Eqs. (1) and (2) as

$$\frac{R_k}{\gamma_R} \geq E_k \gamma_Q. \tag{17}$$

Replacing E_k by its quantile value E_{kq} and assuming a design at the limit of the admissible domain, we obtain

$$R_k = E_{kq} \gamma_Q \gamma_R. \tag{18}$$

Since the characteristic value of the capacity R_k is defined as the 5% quantile, the mean value of R is obtained from the condition

$$F_R^{-1}(0.05) = E_{kq} \gamma_Q \gamma_R. \tag{19}$$

Here, F_R^{-1} is the inverse CDF of R. As evident from Eqs. (18) and (19), the characteristic value of R, and consequently its mean value, are a function of the selected quantile q.

3.4 *Reliability assessment*

The reliability associated with a given quantile value q can be determined by means of the classical structural reliability methods. The load effect E is given by Eq. (14) and it follows that the limit state function describing failure is

$$g(R, Y, Z_m) = R - Y Z_m \tag{20}$$

Here, MCS is applied for determining the probability of failure

$$\Pr(F) = \Pr[g(R, Y, Z_m) \leq 0] \tag{21}$$

and the corresponding reliability index

$$\beta_k^{(T,q)} = \Phi^{-1}[\Pr(F)], \tag{22}$$

with Φ^{-1} being the inverse of the standard Normal cumulative probability function.

4 NUMERICAL INVESTIGATIONS

Numerical investigations are carried out for 18 load effects and different lengths of simulation T. Because only one simulation of total length $T = 30$ min is available for each load effect, the shorter time series are obtained by taking parts of these simulations.

The results for the reference solutions $\beta_k^{(\infty)}$ are obtained by applying the MLE and hypothetically assuming that the covariance of the estimator is zero. For the time series shown in Fig. 2, the resulting values of the characteristic load effect E_{kq} and the corresponding reliability indexes are presented in Fig. 5. The reference reliability index $\beta_k^{(\infty)}$ is also presented. For this case, the necessary quantile to achieve the reference reliability $\beta_k^{(\infty)} = 2.6$ is 0.9.

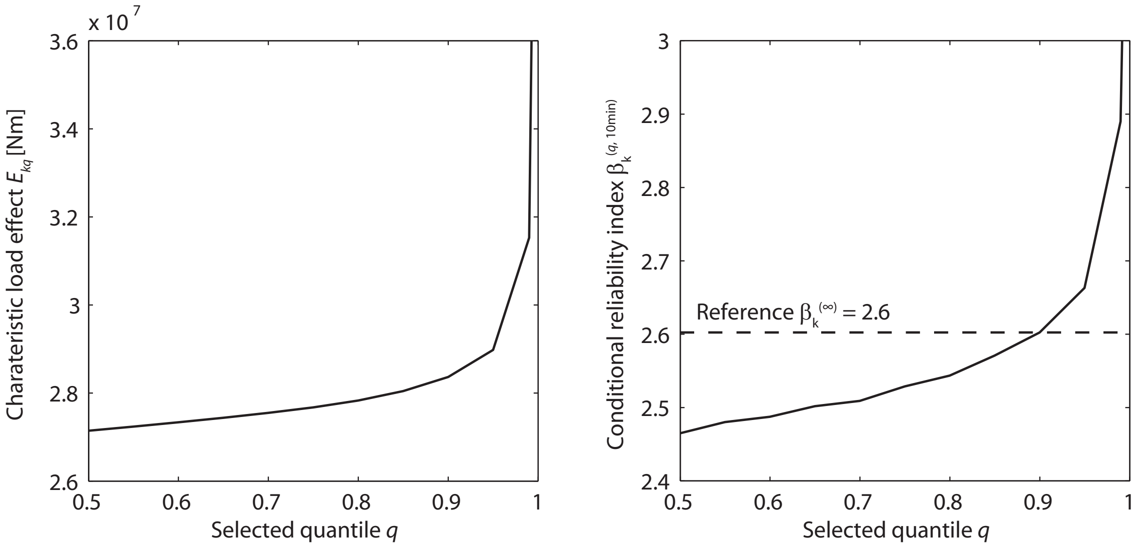

Figure 7. Characteristic value of the load effect and corresponding conditional reliability index $\beta_k^{(q,10\text{min})}$ as a function of the selected quantile value q, for a 10 min time series taken from the one in Fig. 2. The reference reliability index $\beta_k^{(\infty)} = 2.6$ is the value that would be obtained with an infinite time series. The required minimum quantile for this case is 0.9.

A summary of the resulting required quantiles as calculated for the 18 load effects and different durations of the FSI simulation is presented in Fig. 6. The computations show that the necessary quantile q is a function of the length of the simulation: the necessary q decreases with increasing length of the time series.

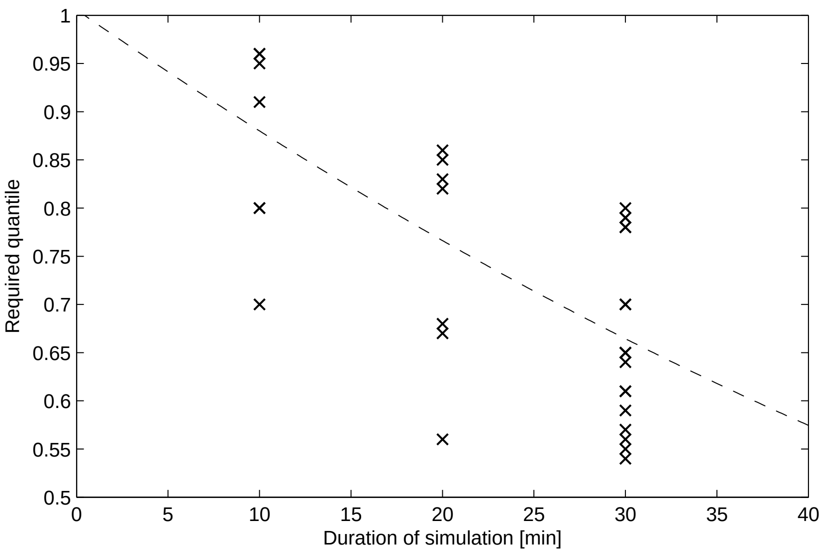

Figure 8. Required quantile values calculated for different load effects and different durations of FSI simulation. The dashed line shows the trend.

The resulting conditional reliability index is in the range 2.4 – 2.7. It is noted that this value is low. However, the reliability of structures subject to predominantly wind load designed according to Eurocode is known to be lower than for structures subject to other loads (JCSS 1996).

Based on the results of this study, it was decided to apply a quantile value of 0.95 and 30 min time series for the design, which is on the conservative side.

5 SUMMARY AND CONCLUSION

Due to computational limitations, the length of time-domain FSI simulations is limited. When estimating the maximum load effect, the uncertainty arising from the limited duration of the load effect should be taken into account. In this paper, we present an engineering approach to dealing with this problem: The statistical uncertainty arising from the limited data is estimated and quantified using a Bayesian approach. It is then proposed to use a quantile value with respect to this uncertainty in the design. The necessary quantile value of the maximum load effect is determined by requiring that the reliability achieved with this quantile value is equal to the reliability that would be achieved when complete information was available (corresponding to an infinite time series). This quantile value must be applied on top of other safety factors and characteristic values; in particular, it does not address the uncertainty in determining the characteristic wind field. The approach is applicable to other problems involving the estimation of extreme actions and load effects on structures based on limited time series, whether they arise from numerical computations or from observations.

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